Constant Bearing Pursuit on Branching Graphs

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Motivation

Some tasks/missions can be accomplished more effectively by groups/collectives of autonomous agents

- Environmental Monitoring
- Search & Rescue
- Intelligence, Surveillance & Reconnaissance (ISR)

A swarm of intelligent 3D printed aquatic surface robots from the University of Lisbon.
Snapshot from: http://www.3ders.org/

Collective control uses local/pairwise interactions to generate desired global motions

- Efficiency
- Robustness

ONR's demonstration of CARACaS (Control Architecture for Robotic Agent Command and Sensing) in August 2014.
http://www.onr.navy.mil/Media-Center/Press-Releases/2014/ autonomous-swarm-boat-unmanned-caracas.aspx

Pursuit Based Collectives

- Pursuit interactions as a building block for collective control

UW-Milwaukee Bio Sciences

STARFLAG project

- Cyclic Constant Bearing (CB) pursuit can generate circling, spiraling & rectilinear motions

[Galloway, Justh, Krishnaprasad - Proc.RoyalSoc.A'13.]

- Limitation 1: Location and Size of the formation depends on initial conditions

  Our earlier work addressed this issue by introducing an external beacon.

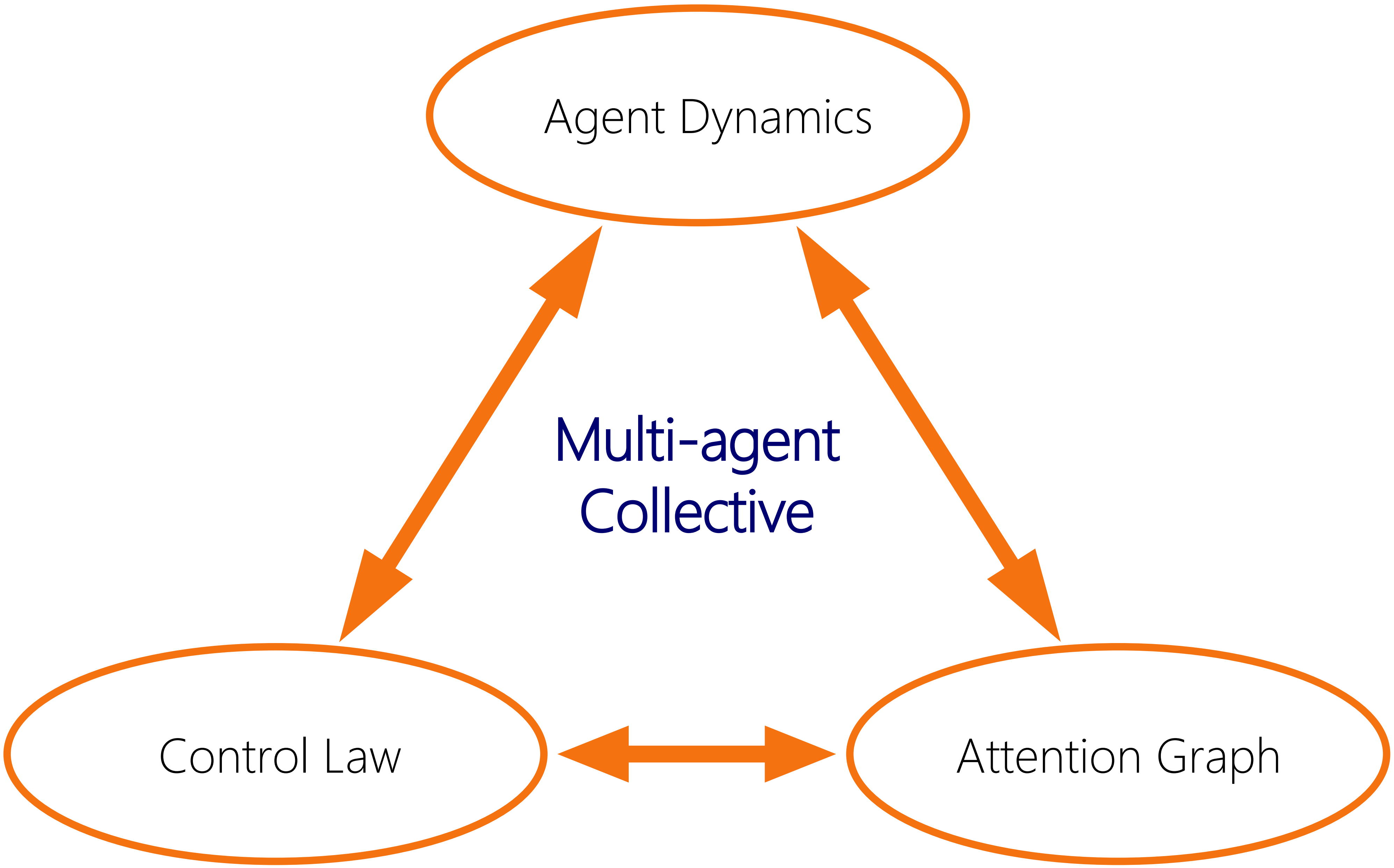
  [Galloway, Dey – ACC'15, ACC'16, Automatica'18]

- Limitation 2: In real applications one may face the need to add more agents to an existing collective, or some of the agents may want to leave the collective prematurely.

Outline

- System Model
- Shape Space and Relative Equilibria
- Motion in the Pure Shape Space
- Stability Analysis for a Special Case

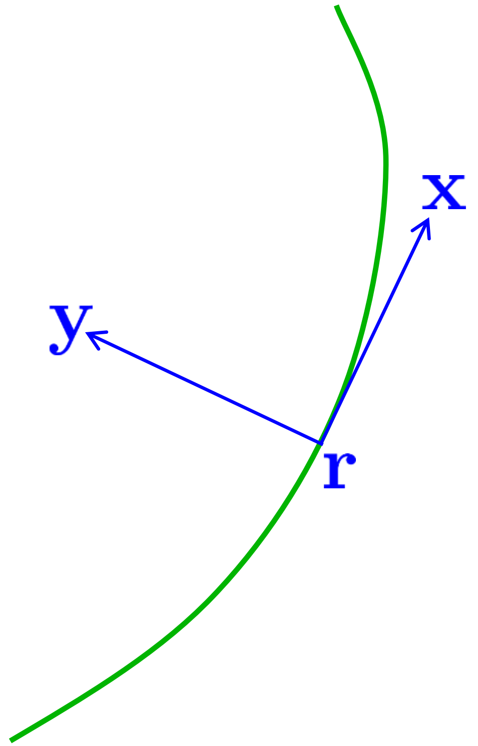
Building Blocks of a Multi-agent Collective



Agents as Self-steering Particles

- Position vector: $\mathbf{r}$
- Natural Frenet frame: $[\mathbf{x} \quad \mathbf{y}]$

- Unit tangent vector: $\mathbf{x} = \dfrac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$
- Speed of the trajectory: $\nu = |\dot{\mathbf{r}}|$



❑ Agent Dynamics:

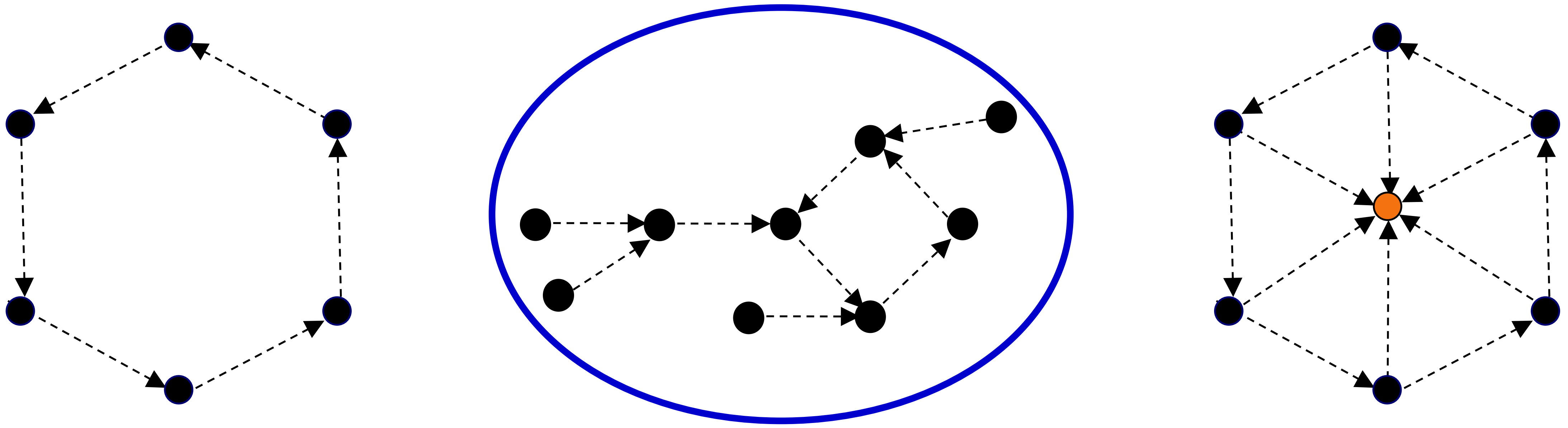
$$\begin{aligned}\dot{\mathbf{r}}(t) &= \nu(t)\mathbf{x}(t)\\ \dot{\mathbf{x}}(t) &= \nu(t)u(t)\mathbf{y}(t)\\ \dot{\mathbf{y}}(t) &= -\nu(t)u(t)\mathbf{x}(t)\end{aligned}$$

Without loss of generality $(\nu \equiv 1)$

$$\begin{aligned}\dot{\mathbf{r}} &= \mathbf{x}\\ \dot{\mathbf{x}} &= u\mathbf{y}\\ \dot{\mathbf{y}} &= -u\mathbf{x}\end{aligned}$$

Attention Graph

- Individual agents are perceived as vertices in a directed graph: $\mathcal{G} = (\mathcal{N}, \mathcal{A})$
  - Node Set: $\mathcal{N} = \{1, 2, \cdots, n\}$
  - Arc Set: $\mathcal{A} = \Big\{(i,j) \in \mathcal{N} \times \mathcal{N} \Big| \text{Agent } i \textit{ pays attention } \text{to Agent } j\Big\}$

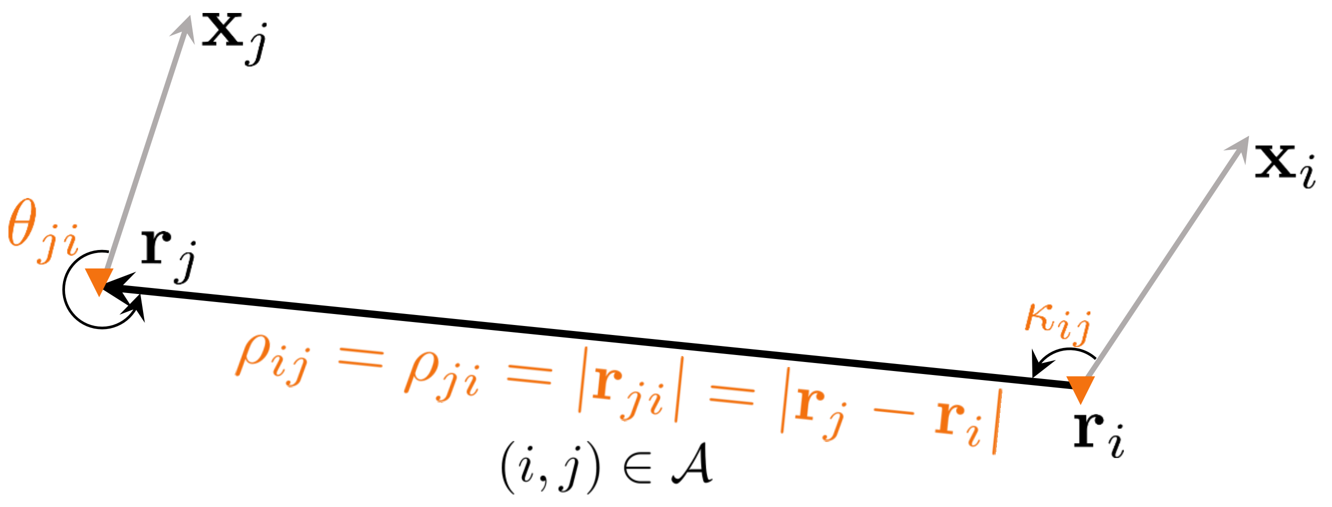


Cycle with Branches

Advantages of Cycles with Branches

- A subset of informed (or well-equipped) agents can form a “base cycle” which others can join or leave at will.
- Resulting dynamics are “triangular”, i.e. agents on the cycle are not affected by the agents on the branches.
- Applies to a very broad class of systems; in particular, any weakly connected attention graph wherein each agent pursues exactly one other agent can be studied in this framework.

Shape Variables with Constraints



- Scalar shape variables describe the relative configuration (position and heading direction) between individual agents.
- To address over parametrization, we include additional constraints along the cycle.

- Cycle Closure Constraints:

$$R\left(\sum_{i=1}^{n}(\pi + \alpha_i - \theta_{i,i-1})\right) = \mathbb{1}$$

$$\sum_{i=1}^{n} \rho_{i,i+1} R\left(\sum_{j=1}^{i}(\pi + \alpha_j - \theta_{j,j-1})\right) = 0$$

$$R(\vartheta) = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}$$

Planar counterclockwise rotation by an angle ϑ

Constant Bearing Pursuit Law

$$u^i_{CB} = \mu_i \left(R(\alpha_i)\mathbf{y}_i \cdot \frac{\mathbf{r}_{ji}}{|\mathbf{r}_{ji}|} \right) - \frac{1}{|\mathbf{r}_{ji}|} \left(\frac{\mathbf{r}_{ji}}{|\mathbf{r}_{ji}|} \cdot R(\pi/2)\dot{\mathbf{r}}_{ji} \right)$$

$$u^i_{CB} = \mu_i \sin(\kappa_{ij} - \alpha_i) + \frac{1}{\rho_{ij}} (\sin \kappa_{ij} + \sin \theta_{ji})$$

Lemma: Define the constant bearing pursuit manifold $M_{CB(\boldsymbol{\alpha})}$ as

$$M_{CB(\boldsymbol{\alpha})} = \left\{ (\kappa_{ij}, \theta_{ji}, \rho_{ij}) \middle| (i,j) \in \mathcal{A}, \rho_{ij} > 0, \cos(\kappa_{ij} - \alpha_i) = 1 \right\}.$$

Then $M_{CB(\boldsymbol{\alpha})}$ remains invariant under the CB pursuit law. In addition, when μ_i's are positive, system trajectories will asymptotically approach $M_{CB(\boldsymbol{\alpha})}$ as $t \mapsto \infty$.

Shape Dynamics on the CB-manifold

- $\mathcal{A} = \{(1,2),(2,3),\ldots,(n-1,1)\} \cup \{(n,1)\}$ (Cycle with single branch)
- As the state trajectories converge to the CB pursuit manifold for positive values of the feedback gain, we consider the dynamics to be restricted to the CB pursuit manifold for further analysis.

$$\dot{\theta}_{i+1,i} = \frac{1}{\rho_{i,i+1}}(\sin\alpha_i + \sin\theta_{i+1,i}) - \frac{1}{\rho_{i+1,i+2}}(\sin\alpha_{i+1} + \sin\theta_{i+2,i+1})$$

$$\dot{\rho}_{i,i+1} = -\cos\alpha_i - \cos\theta_{i+1,i}$$

$$i = 1,2,\ldots,n-1$$

$$\dot{\theta}_{1n} = \frac{1}{\rho_{n1}}(\sin\alpha_n + \sin\theta_{1n}) - \frac{1}{\rho_{12}}(\sin\alpha_1 + \sin\theta_{21})$$

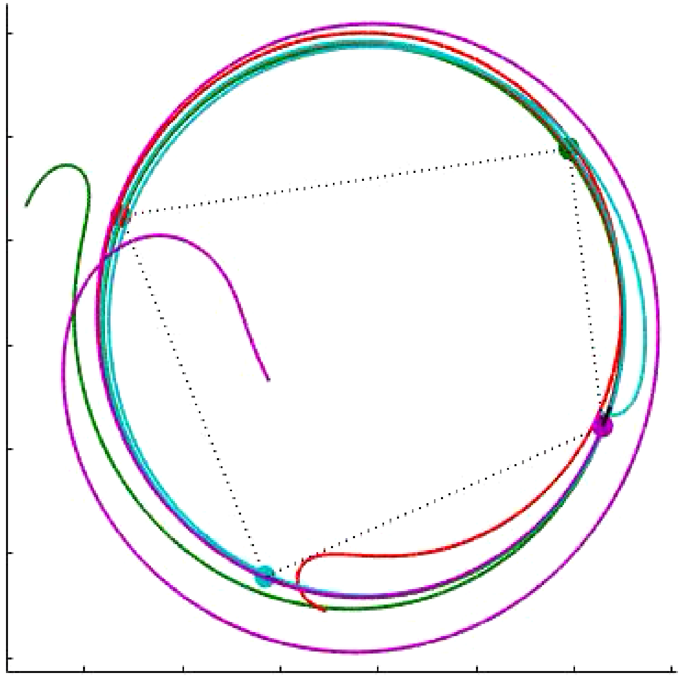
$$\dot{\rho}_{n1} = -\cos\alpha_n - \cos\theta_{1n}$$

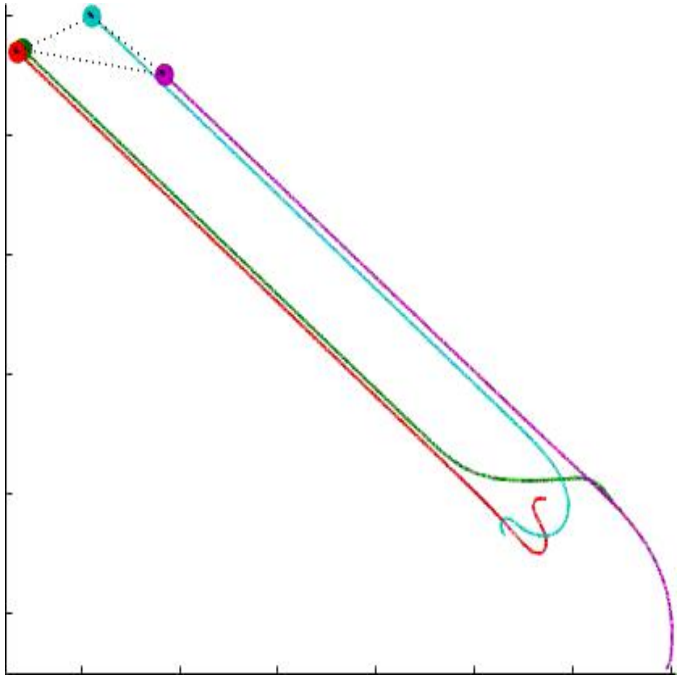
Dynamics is Triangular

Relative Equilibrium

Rectilinear Equilibrium *if and only if*

- $\alpha_i = \pi + \alpha_{i-1}, \qquad i = 1, \cdots, n-1$

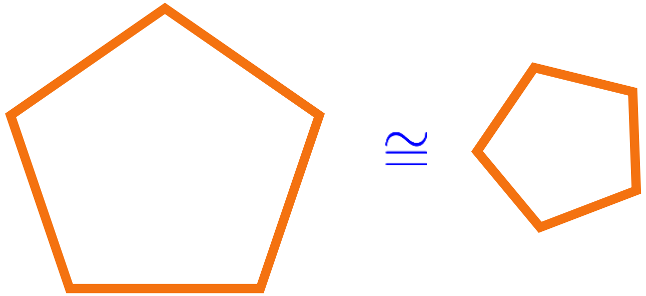




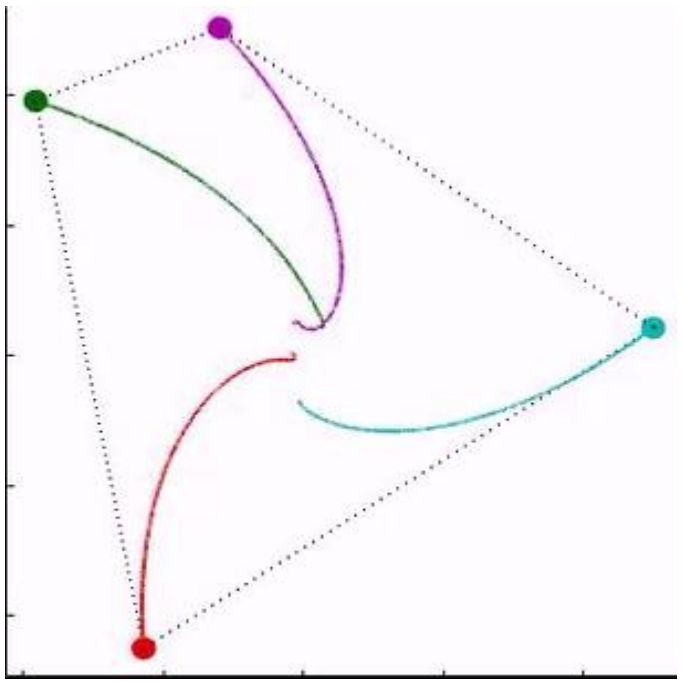
Circling Equilibrium *if and only if*

- $\sin(\alpha_i)\sin(\alpha_j) > 0, \ (i,j) \in \mathcal{A}$
- $\sum_{i=1}^{n-1} (\alpha_i) = 2k\pi$

Pure Shape Equilibrium – Spiral Motion



- Change of variables allow us to separate out the scale/size of the formation and its pure shape.
- Relative equilibria are pure shape equilibria as well.
- In addition, pure shape equilibria also include spiraling motions.



Pure Shape Equilibrium *if and only if*

$$\sin(\alpha_i - \tau_k)\sin(\alpha_j - \tau_k) > 0, \quad (i,j) \in \mathcal{A}$$

$$\tau_k \triangleq \sum_{i=1}^{n-1} \frac{\alpha_i}{n-1} - \frac{k\pi}{n-1}$$

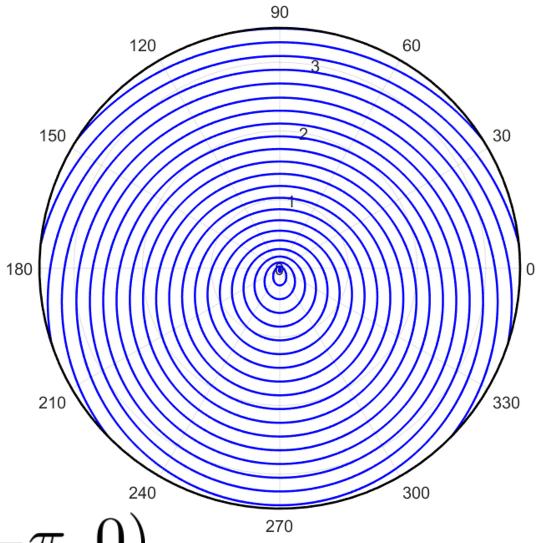
$$k \in \{0, 1, \ldots, n-2\}$$

Periodic Orbit in Pure Shape Space

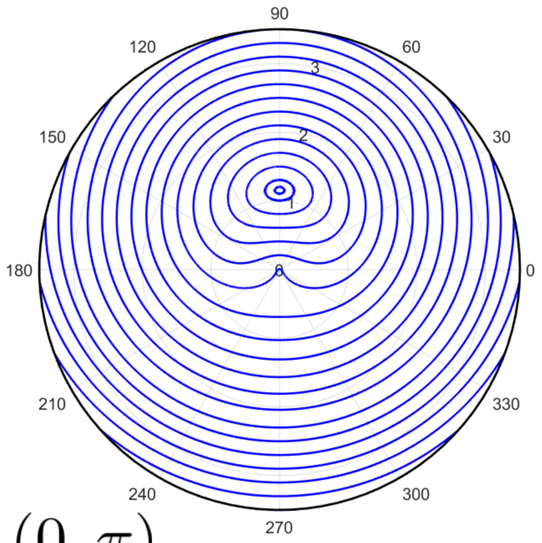
Circling Equilibrium

$+$

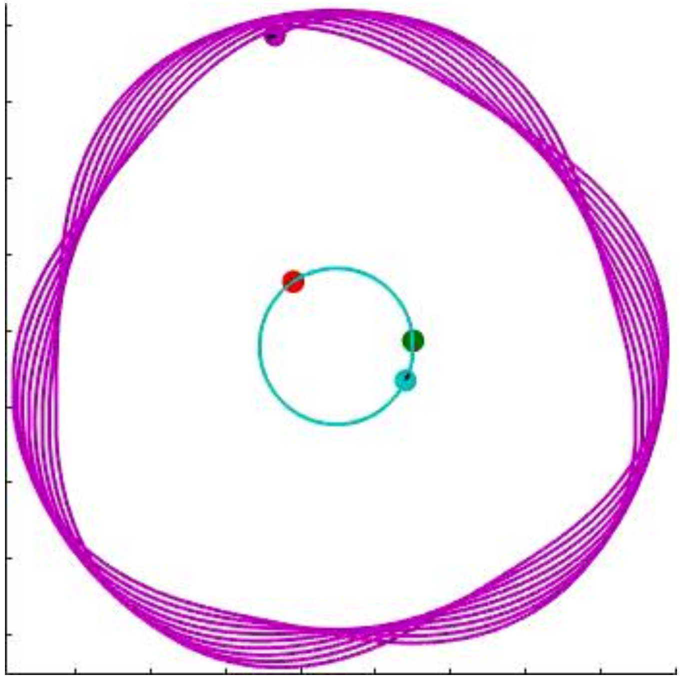
$\alpha_n = \pi/2$

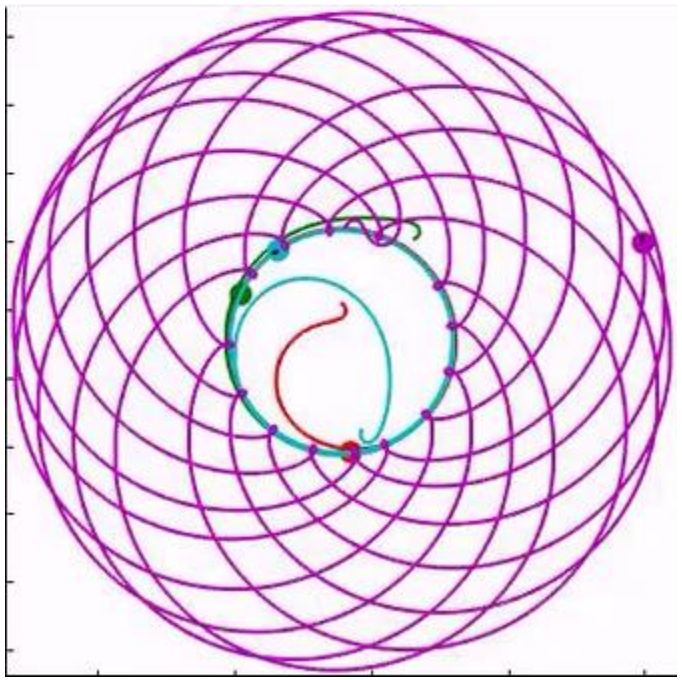


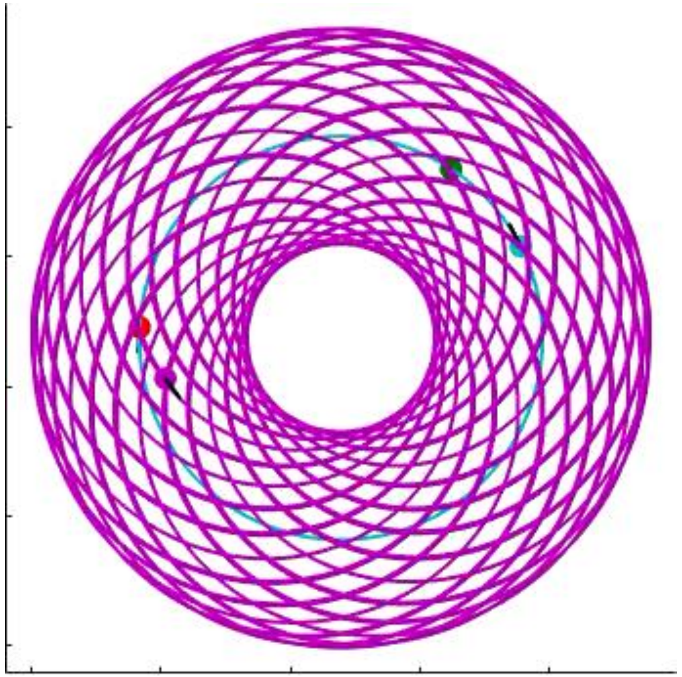
$\alpha_1 \in (-\pi, 0)$



$\alpha_1 \in (0, \pi)$







Stability for a 3-agent system

Rectilinear Equilibrium

- There exists a continuum of equilibria.
- Conjecture:

$$\cos\alpha_3 > 0 \Rightarrow \text{Attractivity}$$

Circling Equilibrium

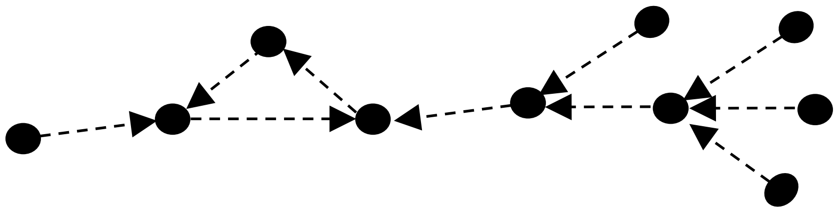
- $\cos\alpha_3 > 0 \Rightarrow \text{Stability}$
- $\cos\alpha_3 < 0 \Rightarrow \text{Instability}$

Pure Shape Equilibrium

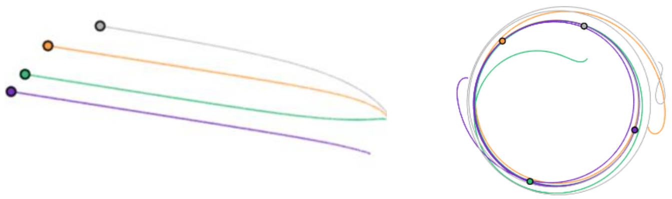
- $2\cos(\alpha_1 + \alpha_2 - \alpha_3) + \cos(\alpha_3) < 0 \Rightarrow \text{Stability}$
- $2\cos(\alpha_1 + \alpha_2 - \alpha_3) + \cos(\alpha_3) > 0 \Rightarrow \text{Instability}$

Conclusion

- Introduced an attention graph with broader scope and better protection against agent failure.



- Relative equilibria corresponds to rectilinear and circling formation.



- Equilibria and periodic orbits exist in the pure shape space as well.