Station Keeping
via
Beacon-referenced Cyclic Pursuit

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Motivation

Some tasks/missions can be accomplished more effectively by groups (collectives) of autonomous agents

- Search & Rescue
- Environmental Sensing
- Battlefield surveillance

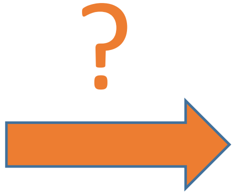
Collective control uses **local interactions** to generate desired **global motions**

US Coast Guard helicopter rescue demonstration" by Brandon Weeks - Own work. Licensed under CC BY 3.0 via Wikimedia Commons - https://commons.wikimedia.org/wiki/File:US_Coast_Guard_helicopter_rescue_demonstration.jpg#/media/File:US_Coast_Guard_helicopter_rescue_demonstration.jpg

Pursuit Based Collectives

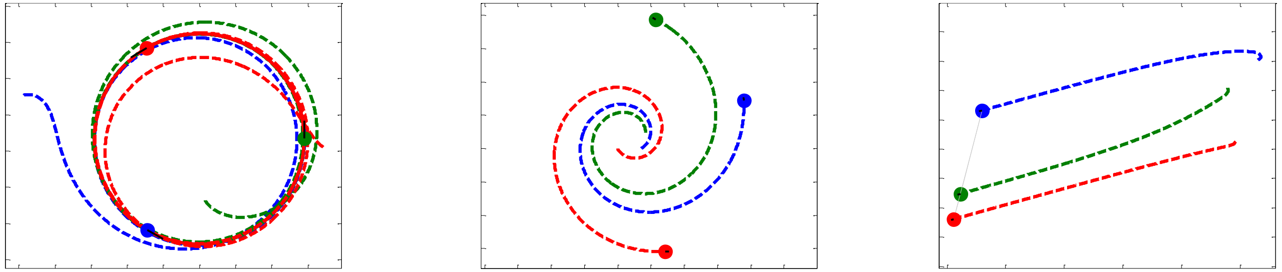
Pursuit interactions as a building block for collective control

UW-Milwaukee Bio Sciences



STARFLAG project, INFM-CNR

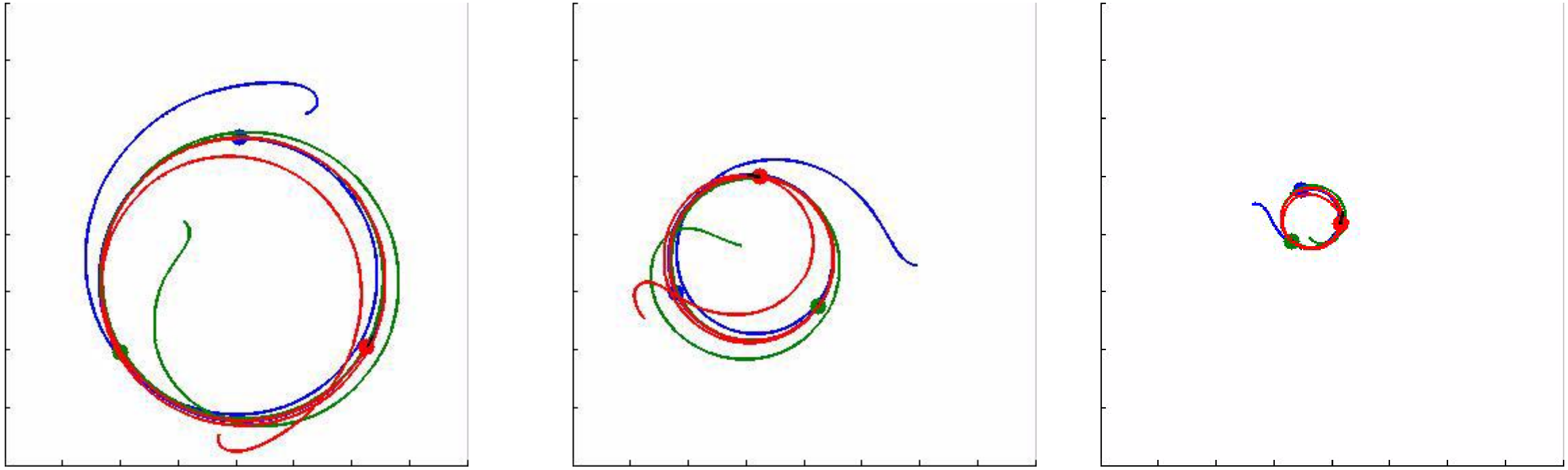
Cyclic Constant Bearing (CB) pursuit can generate circling, spiraling, & rectilinear motions



Galloway, Justh, Krishnaprasad, “Symmetry and Reduction in Collectives: Cyclic Pursuit Strategies”, Proc. R. Soc. A, 2013.

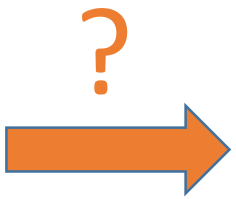
Pursuit Based Collectives

Cyclic CB Pursuit
- Same control parameters
- Different initial conditions



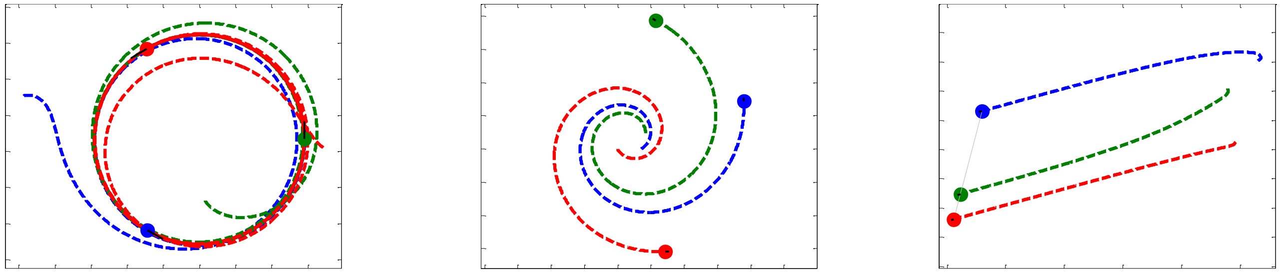
Pursuit Based Collectives

Pursuit interactions as a building block for collective control



Cyclic Constant Bearing (CB) pursuit can generate circling, spiraling, & rectilinear motions

- Limitation: center and radius of circular orbit depends on initial conditions



Galloway, Justh, Krishnaprasad, "Symmetry and Reduction in Collectives: Cyclic Pursuit Strategies", Proc. R. Soc. A, 2013.

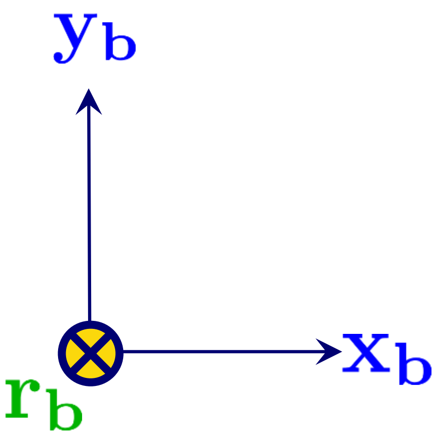
Current work introduces an external reference (i.e. a beacon) sensed by all agents

- Could represent a distress signal, food source, etc.

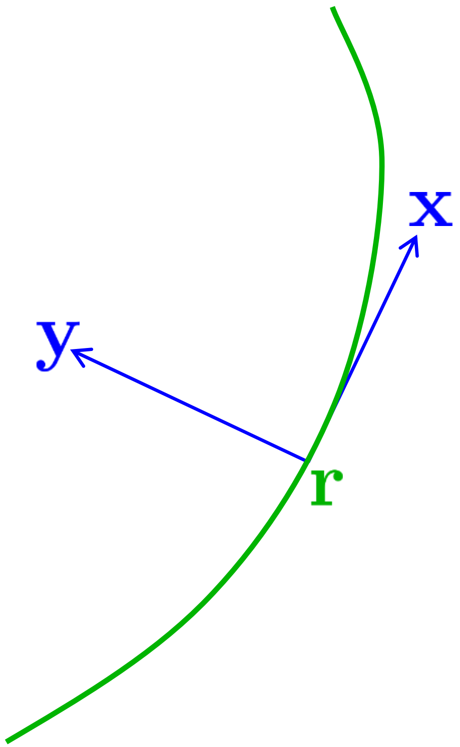
Overview

- Modeling
- Reduction to Shape Space
- Relative Equilibrium
- A special case: 2-agent system
- Implementation in a Lab Environment
- Future Directions

Modeling: Individual Agents



- Beacon modeled as a particle with zero speed and arbitrary frame orientation



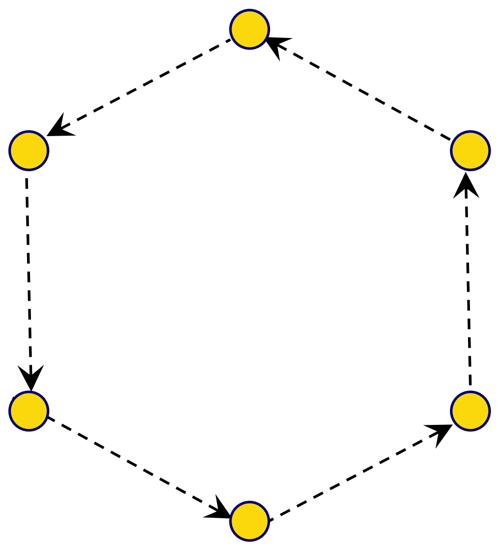
- Position vector: $\mathbf{r}$
- Natural Frenet frame: $[\mathbf{x} \;\; \mathbf{y}]$
- Unit tangent vector: $\mathbf{x} = \dfrac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$
- Speed of the trajectory: $\nu = |\dot{\mathbf{r}}|$

- Agent Dynamics:

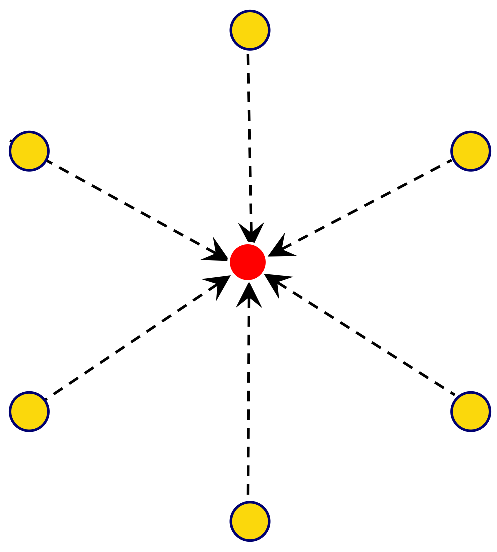
$$
\begin{aligned}
\dot{\mathbf{r}}(t) &= \nu(t)\mathbf{x}(t) \\
\dot{\mathbf{x}}(t) &= \nu(t)u(t)\mathbf{y}(t) \\
\dot{\mathbf{y}}(t) &= -\nu(t)u(t)\mathbf{x}(t)
\end{aligned}
$$

Modeling: Attention Graph

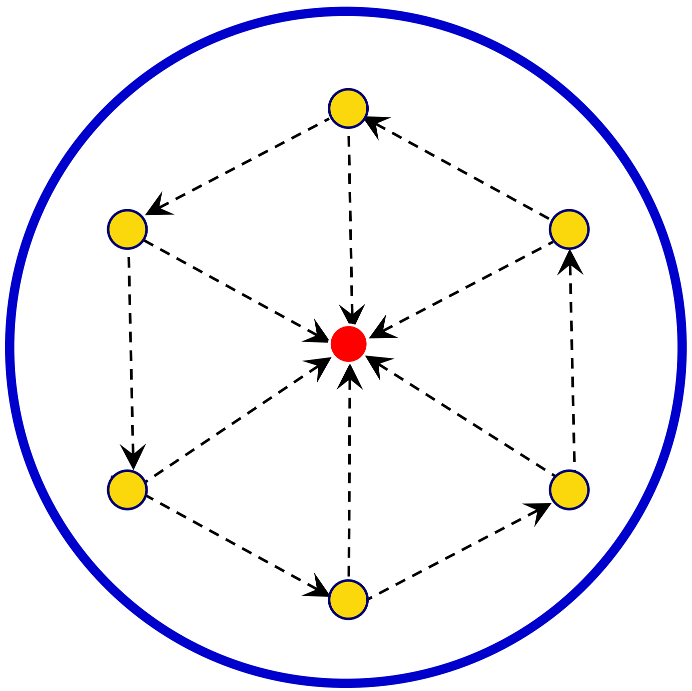
- Individual agents are perceived as vertices in a *directed graph*: $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
  - **Vertex Set**: $\mathcal{V} = \{1, 2, \cdots, n\}$
  - **Edge Set**: $\mathcal{E} = \left\{ (i,j) \in \mathcal{V} \times \mathcal{V} \middle| \text{Agent } i \textit{ pays attention} \text{ to Agent } j \right\}$



Cycle



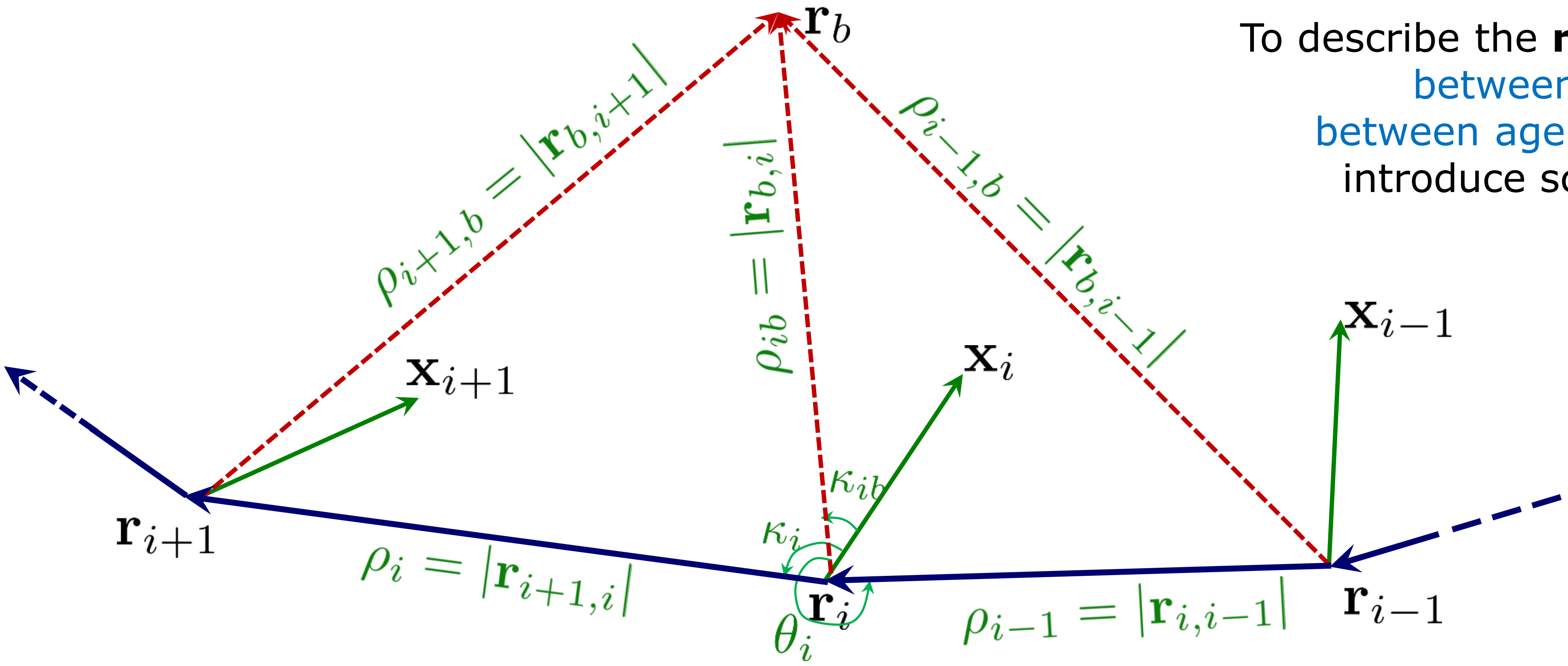
Star



Cycle with Spokes

Reduction to Shape Space (of relative *Position* & *Orientation*)

To describe the **relative configuration** between individual agents, and between agents and the beacon, we introduce scalar **shape variables**.



- **Consistency Condition**

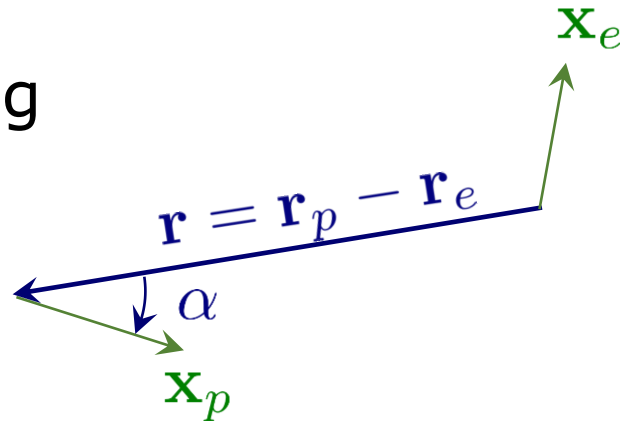
$$\rho_i \mathbb{I}_2 = \rho_{ib} R(\kappa_{ib} - \kappa_i) + \rho_{i+1,b} R(\kappa_{i+1,b} - \theta_{i+1})$$

- **Cycle Closure Constraint**

$$R\Big(\sum_{i=1}^{n} (\pi + \kappa_i - \theta_{i+1})\Big) = \mathbb{I}_2$$

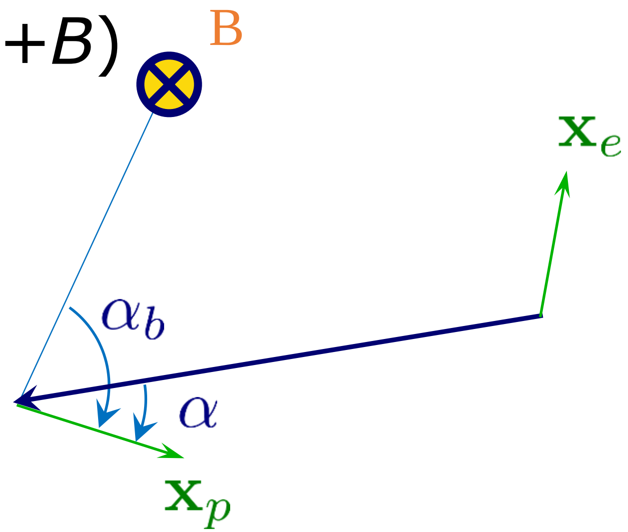
Modeling: Beacon Referenced CB Pursuit

Constant Bearing Pursuit (*CB*)



$$u^i_{CB} = -\mu_i \left(R(\alpha_i)\mathbf{y}_i \cdot \frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|} \right) - \frac{1}{\nu_i |\mathbf{r}_{i,i+1}|} \left(\frac{\mathbf{r}_{i,i+1}}{|\mathbf{r}_{i,i+1}|} \cdot R(\pi/2)\dot{\mathbf{r}}_{i,i+1} \right)$$

Beacon-referenced CB Pursuit (*CB*+*B*)

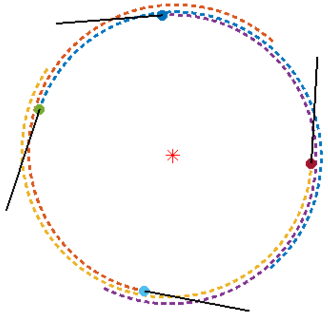


$$u^i_{CB+B} = (1-\lambda)u^i_{CB} - \lambda\mu^b_i \underbrace{\left(R(\alpha_b)\mathbf{y}_i \cdot \frac{\mathbf{r}_{i,b}}{|\mathbf{r}_{i,b}|} \right)}_{\text{References only relative bearing to the beacon}}$$

Weighting factor

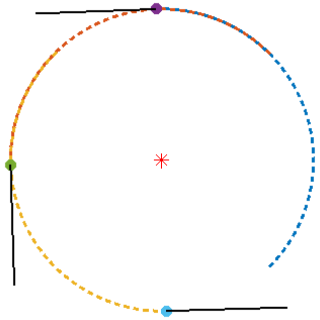
Beacon-referenced Cyclic Pursuit

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$
$$\mathcal{V} = \{1, 2, \cdots, n, b\}$$
$$\mathcal{E} = \Big\{(i, i+1), (i, b) \Big| i = 1, 2, \cdots, n\Big\}$$



Cycle with Spokes + Beacon-referenced CB pursuit law (*CB*+*B*) = Beacon-referenced Cyclic Pursuit

$$u^i_{CB+B} = \lambda \mu_i^b \sin(\kappa_{ib} - \alpha_{ib}) + (1-\lambda)\mu_i \sin(\kappa_i - \alpha_i) + \frac{1-\lambda}{\rho_i}\left(\sin\kappa_i + \frac{\nu_{i+1}}{\nu_i}\sin\theta_{i+1}\right)$$



Closed Loop Shape Dynamics

- **Assumptions**
  - Individual speeds are constant and equal.
  - Controller gains are equal and common for every agent.
  - The bearing angle parameters (*for the beacon*) are equal for every agent.

$$
\begin{aligned}
\dot{\rho}_i &= -\left(\cos\kappa_i + \cos\theta_{i+1}\right)\\
\dot{\kappa}_i &= -\mu\left[(1-\lambda)\sin(\kappa_i-\alpha_i) + \lambda\sin(\kappa_{ib}-\alpha_0)\right] + \frac{\lambda}{\rho_i}\left[\sin\kappa_i + \sin\theta_{i+1}\right]\\
\dot{\theta}_i &= \dot{\kappa}_i - \frac{1}{\rho_i}\left[\sin\kappa_i + \sin\theta_{i+1}\right] + \frac{1}{\rho_{i-1}}\left[\sin\kappa_{i-1} + \sin\theta_i\right]\\
\dot{\rho}_{ib} &= -\cos\kappa_{ib}\\
\dot{\kappa}_{ib} &= \dot{\kappa}_i - \frac{1}{\rho_i}\left[\sin\kappa_i + \sin\theta_{i+1}\right] + \frac{1}{\rho_{ib}}\sin\kappa_{ib}
\end{aligned}
$$

Subject to consistency and closure constraints.

Closed Loop Shape Dynamics

- **Assumptions**
  - Individual speeds are constant and equal.
  - Controller gains are equal and common for every agent.
  - The bearing angle parameters (*for the beacon*) are equal for every agent.

$\dot{\rho}_i =$ Inter-agent range dynamics

$\dot{\kappa}_i =$ Line-of-sight Angle to Pursuee dynamics

$\dot{\theta}_i =$ Line-of-sight Angle to Pursuer dynamics

$\dot{\rho}_{ib} =$ Agent-Beacon range dynamics

$\dot{\kappa}_{ib} =$ Line-of-sight Angle to Beacon dynamics

Subject to consistency and closure constraints.

Relative Equilibrium

$$\dot{\rho}_{ib} = 0 \rightarrow \boxed{\kappa_{ib} = \frac{\pi}{2}}, \quad \boxed{\kappa_{ib} = -\frac{\pi}{2}}$$

$$\dot{\rho}_{i} = 0 \rightarrow \boxed{\theta_{i+1} = \pi - \kappa_i}, \quad \boxed{\theta_{i+1} = \pi + \kappa_i}$$

$$\dot{\kappa}_{ib} = 0 \rightarrow \boxed{\frac{\sin\kappa_{i-1} + \sin\theta_i}{\rho_{i-1}} = \frac{\sin\kappa_i + \sin\theta_{i+1}}{\rho_i}}$$

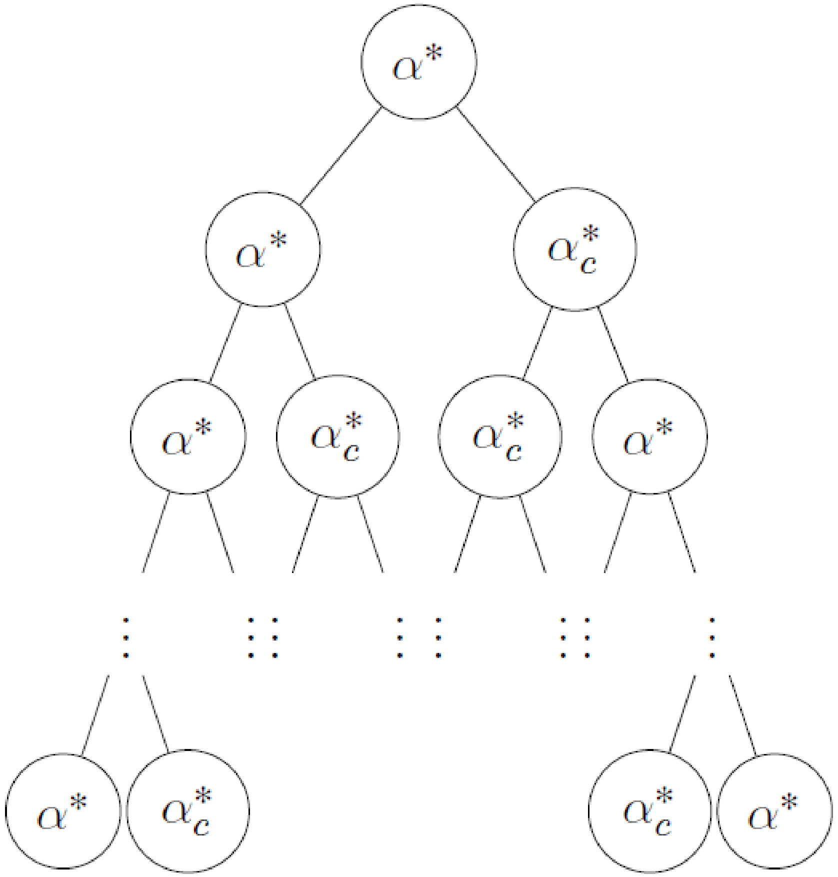
$$\boxed{\frac{\sin\kappa_i + \sin\theta_{i+1}}{\rho_i} = \frac{\sin\kappa_{ib}}{\rho_{ib}}} \leftarrow \dot{\theta}_i = 0$$

$$\dot{\kappa}_i = 0 \rightarrow \boxed{\sin(\kappa_{i+1} - \alpha_{i+1}) = \sin(\kappa_i - \alpha_i)} \qquad i = 1, 2, \cdots, n$$

Relative Equilibrium (Continued)

$$\dot{\kappa}_i = 0 \longrightarrow \boxed{\sin(\kappa_{i+1} - \alpha_{i+1}) = \sin(\kappa_i - \alpha_i)} \qquad i = 1, 2, \cdots, n$$

$$\longrightarrow \kappa_{i+1} - \alpha_{i+1} = \begin{cases} \kappa_i - \alpha_i \\ \pi - (\kappa_i - \alpha_i) \end{cases} \qquad i = 1, 2, \cdots, n$$



#1

#2

#3

⋮

#n

Let $\kappa_1 - \alpha_1 = \alpha^*$.

Our task: find an α^* and appropriate branch such that

1. $\rho_i > 0, \; i = 1, 2, \ldots, n,$
2. $\rho_{ib} \geq 0, \; i = 1, 2, \ldots, n,$
3. consistency conditions are satisfied,
4. cycle closure constraints are satisfied.

Relative Equilibrium (Continued)

Proposition: Consider an n-agent cyclic CB system with beacon, parametrized by μ, λ, and $\{\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_n\}$.

(a) The only possible relative equilibria are **circling equilibria**.

Branch of binary tree

(b) Whenever $\sin(\sum \alpha_i) \neq 0$, a circling equilibrium exists *if and only if* there exists $m \in \mathbb{Z}$ and $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n) \in \{-1, 1\}^n$ such that

(i) the cardinality M of the subset $\{\sigma_i | \sigma_i = 1, i = 1, 2, \ldots, n\}$ satisfies $2M - n \neq 0$,

(ii)
$$\begin{aligned} &\lambda \cos \alpha_0 + (1 - \lambda) \sin \alpha^* > 0, \\ &\sin\left(\alpha^* + \sigma_i \alpha_i\right) > 0, \qquad i = 1, 2, \ldots, n, \end{aligned}$$

Positivity constraints

where α^* is given by $\alpha^* = \left(\dfrac{m + M - n}{2M - n}\right)\pi - \sum_{i=1}^{n} \left(\dfrac{\alpha_i}{2M - n}\right)$.

Relative Equilibrium (Continued)

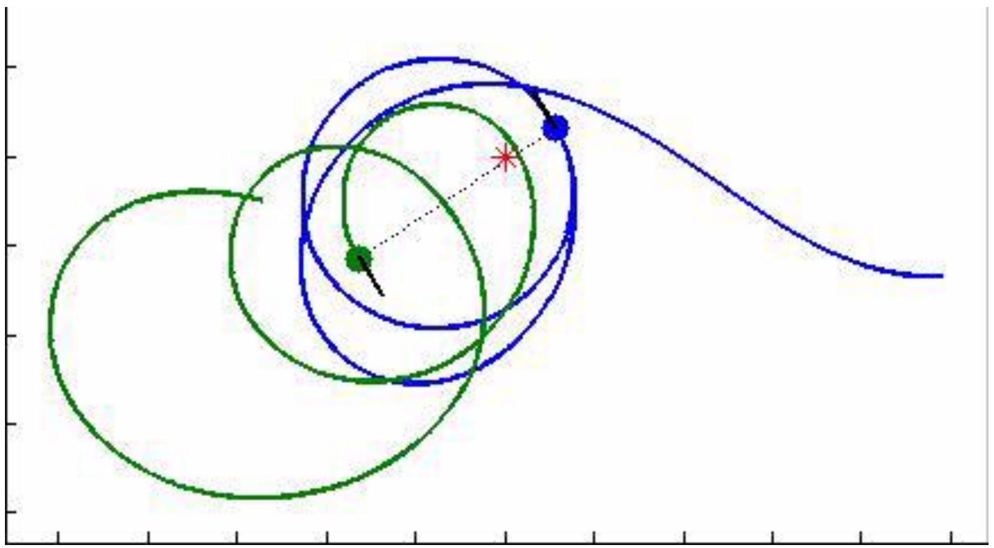
Equilibrium Values for Shape Variables

- $$\kappa_i = \begin{cases} \alpha^* + \alpha_i & \text{if } \sigma_i = +1 \\ \pi - \alpha^* + \alpha_i & \text{if } \sigma_i = +1 \end{cases}$$
- $$\kappa_{ib} = \begin{cases} \pi/2 & \forall i \\ -\pi/2 & \forall i \end{cases}$$
- $$\rho_i = \frac{2 \sin \kappa_{ib} \sin \kappa_i}{\mu \left(\frac{1}{\lambda} - 1 \right) \sin \kappa_{ib} \sin \alpha^* + \mu \cos \alpha_0}$$
- $$\theta_{i+1} = \pi - \kappa_i$$
- $$\rho_{ib} = \frac{1}{\mu \left(\frac{1}{\lambda} - 1 \right) \sin \kappa_{ib} \sin \alpha^* + \mu \cos \alpha_0}$$

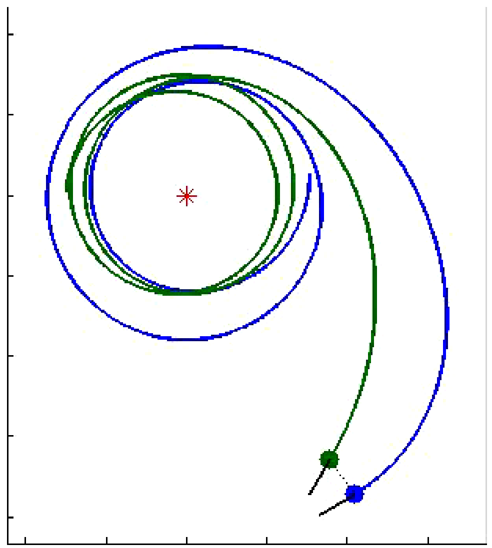
<u>Note</u>: circling equilibrium radius determined by control parameters, not initial conditions

Simulation Results

$$\alpha_1 = \alpha_2 = \frac{\pi}{4}, \quad \alpha_0 = \frac{\pi}{4}$$



$$\alpha_1 = \alpha_2 = \frac{\pi}{4}, \quad \alpha_0 = -\frac{\pi}{3}$$



2-Agent System

System Dynamics

$$\dot{\rho} = -(\cos\kappa_1 + \cos\kappa_2)$$

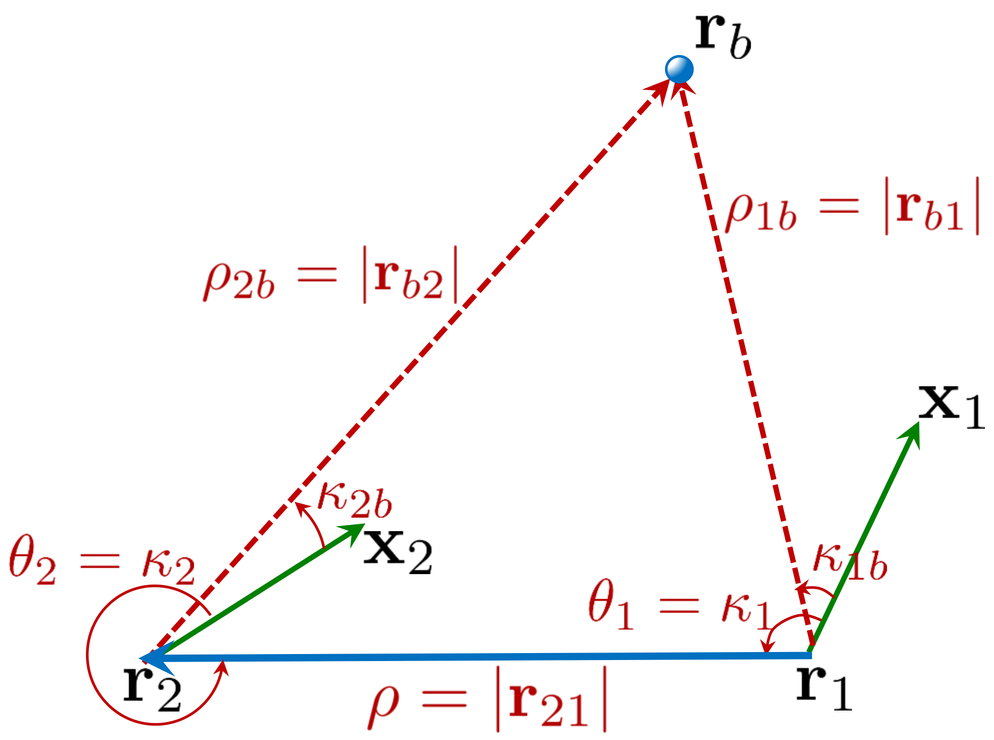
$$\dot{\kappa}_i = -\mu\Big[(1-\lambda)\sin(\kappa_i - \alpha_i) + \lambda\sin(\kappa_{ib} - \alpha_0)\Big] + \frac{\lambda}{\rho}(\sin\kappa_1 + \sin\kappa_2)$$

$$\dot{\rho}_{ib} = -\cos\kappa_{ib}$$

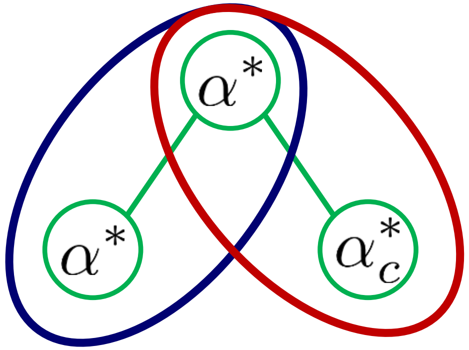
$$\dot{\kappa}_{ib} = \dot{\kappa}_i - \frac{1}{\rho}(\sin\kappa_1 + \sin\kappa_2) + \frac{1}{\rho_{ib}}\sin\kappa_{ib}$$

Constraints

$$\rho\mathbb{I}_2 - \rho_{1b}R(\kappa_{1b} - \kappa_1) - \rho_{2b}R(\kappa_{2b} - \kappa_2) = 0$$



2-Agent System: Circling Equilibrium



- $\alpha_1 + \alpha_2 \neq k\pi, \quad k \in \mathbb{Z}$

$$\boxed{\kappa_i - \alpha_i = \alpha^* = m\left(\frac{\pi}{2}\right) - \alpha^+} \qquad i = 1, 2; \quad m \in \mathbb{Z}$$

  - $\operatorname{sgn}(\sin \kappa_1) = \operatorname{sgn}(\sin \kappa_2)$ $\begin{cases} m = 1 \quad :: & \kappa_1 = \dfrac{\pi}{2} + \alpha^-, \quad \kappa_2 = \dfrac{\pi}{2} - \alpha^- \\ m = -1 :: & \kappa_1 = -\dfrac{\pi}{2} + \alpha^-, \quad \kappa_2 = -\dfrac{\pi}{2} - \alpha^- \end{cases}$

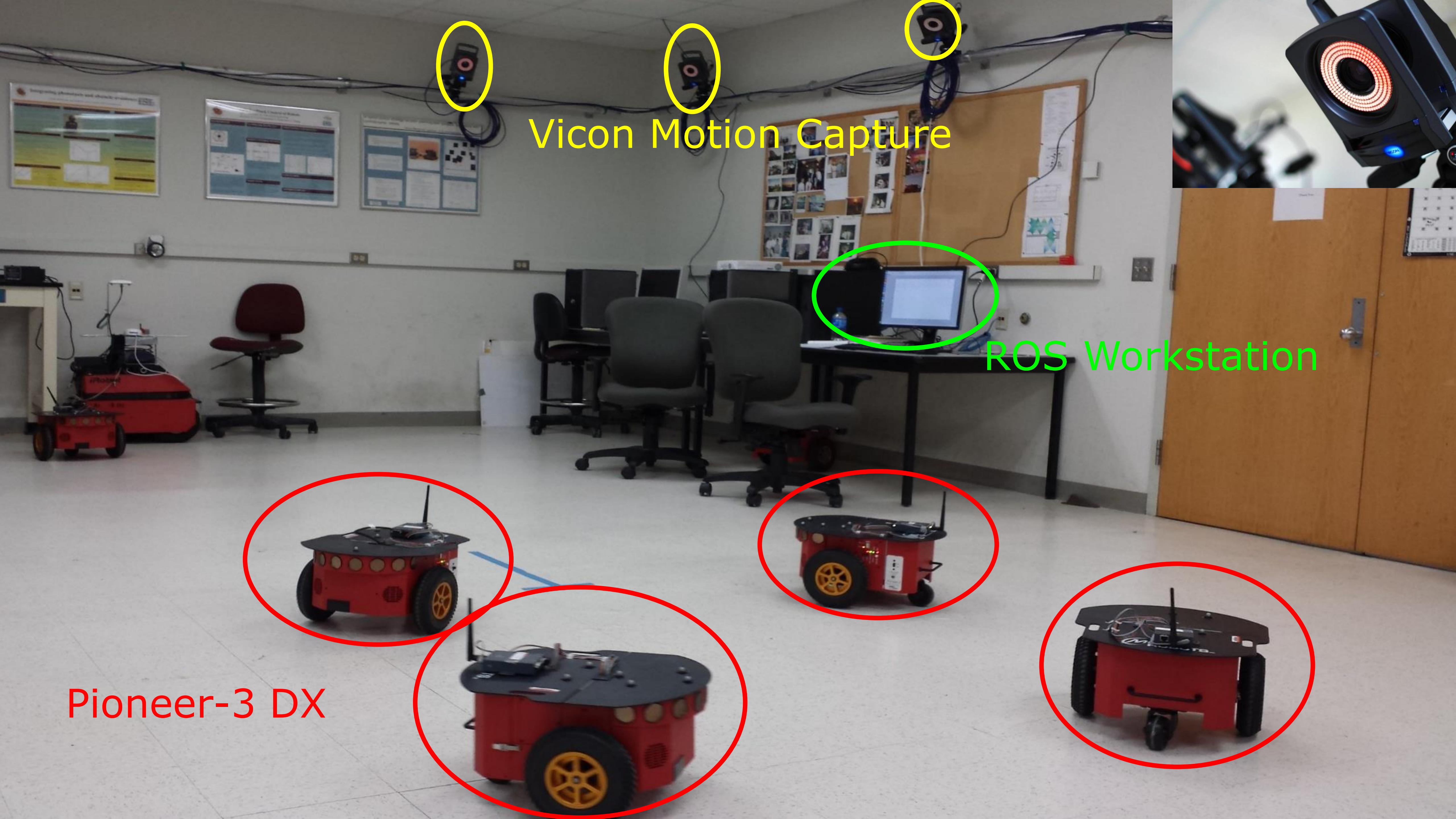
$$\alpha^+ \triangleq \frac{\alpha_1 + \alpha_2}{2}, \qquad \alpha^- \triangleq \frac{\alpha_1 - \alpha_2}{2}$$

- $\alpha_1 + \alpha_2 = k\pi, \quad k \in \mathbb{Z}$
  - A continuum of equilibria exist.

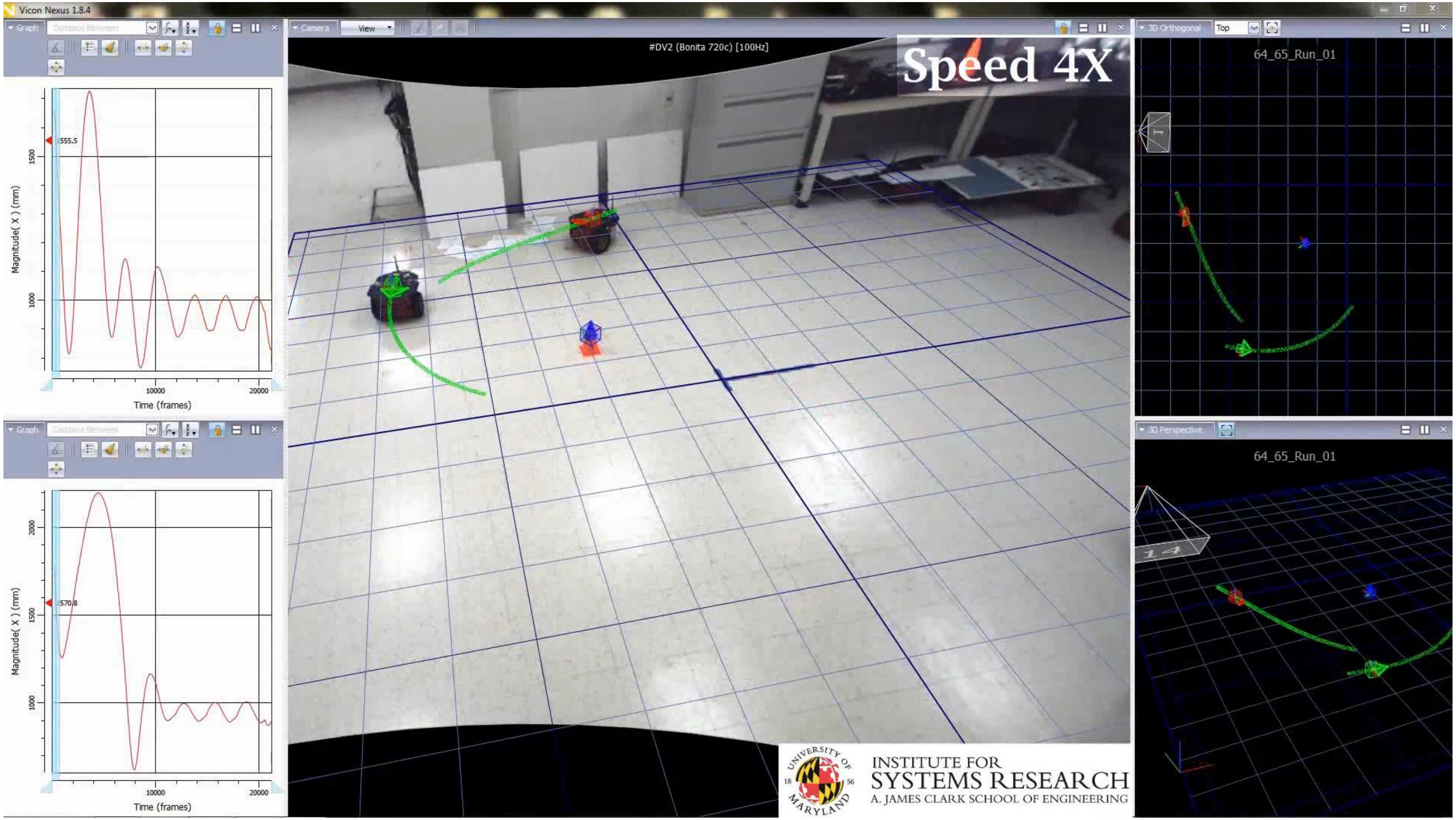
2-Agent System: Stability

- Stability analysis is local in nature.

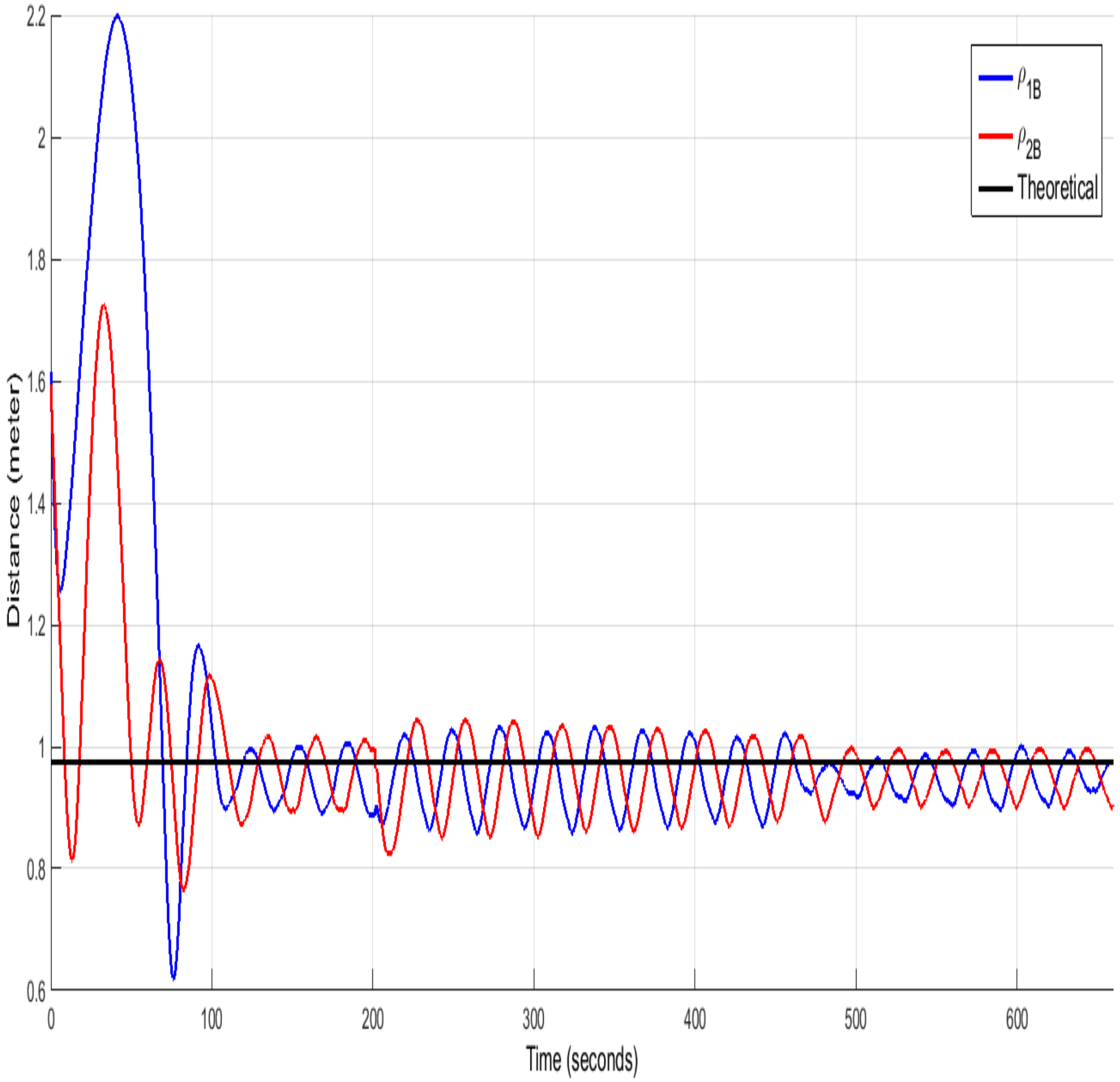
	CCW	CW
	$\kappa_{ib} = \frac{\pi}{2}$	$\kappa_{ib} = -\frac{\pi}{2}$
$m = 1$	$\sin \alpha_0 > 0$ $\sin \alpha^+ > 0$	$\sin \alpha_0 < 0$ $\sin \alpha^+ > 0$
$m = -1$	$\sin \alpha_0 > 0$ $\sin \alpha^+ < 0$	$\sin \alpha_0 < 0$ $\sin \alpha^+ < 0$

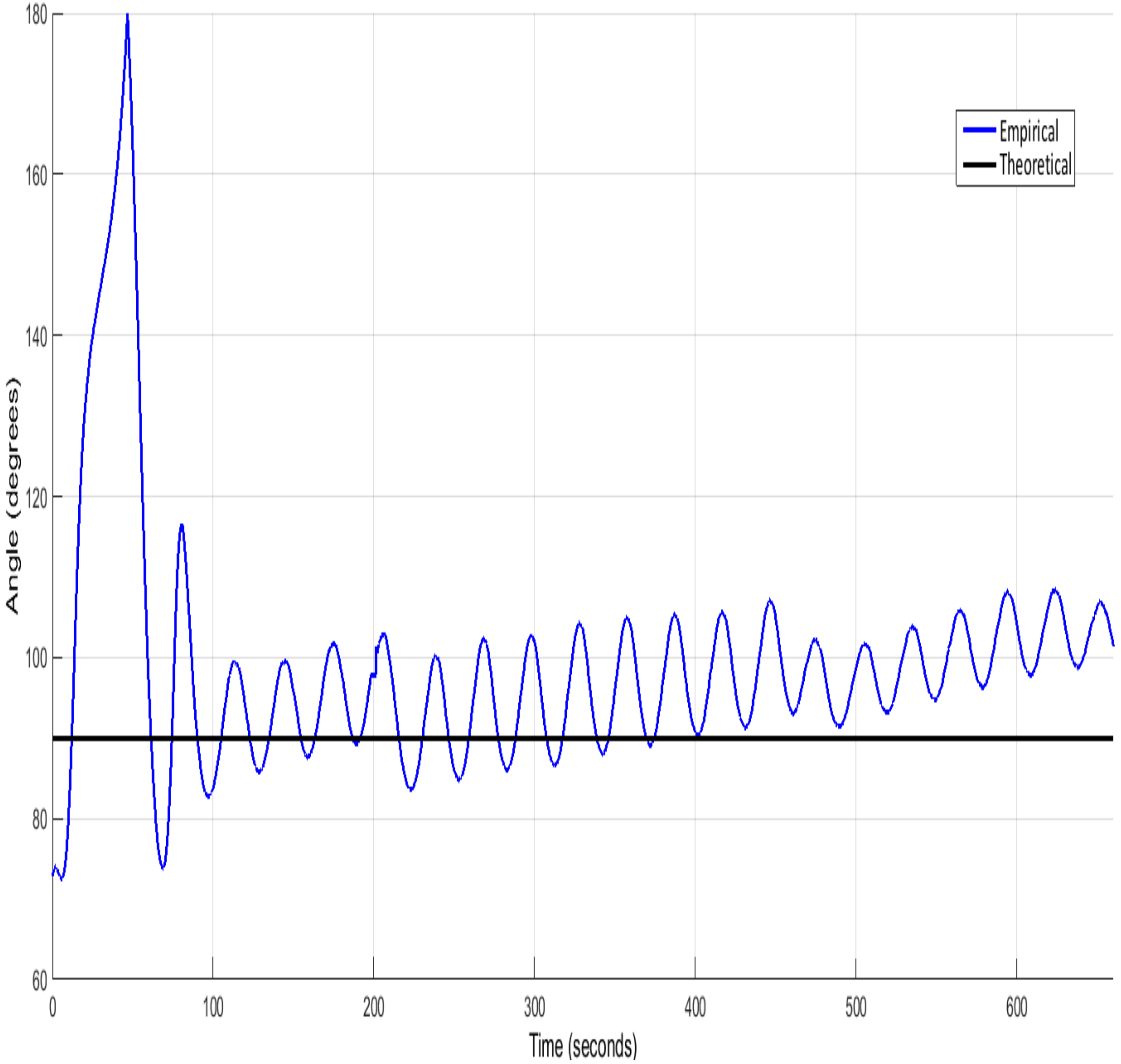

Vicon Motion Capture
ROS Workstation
Pioneer-3 DX

Implementation on Mobile Robots (2 robots)

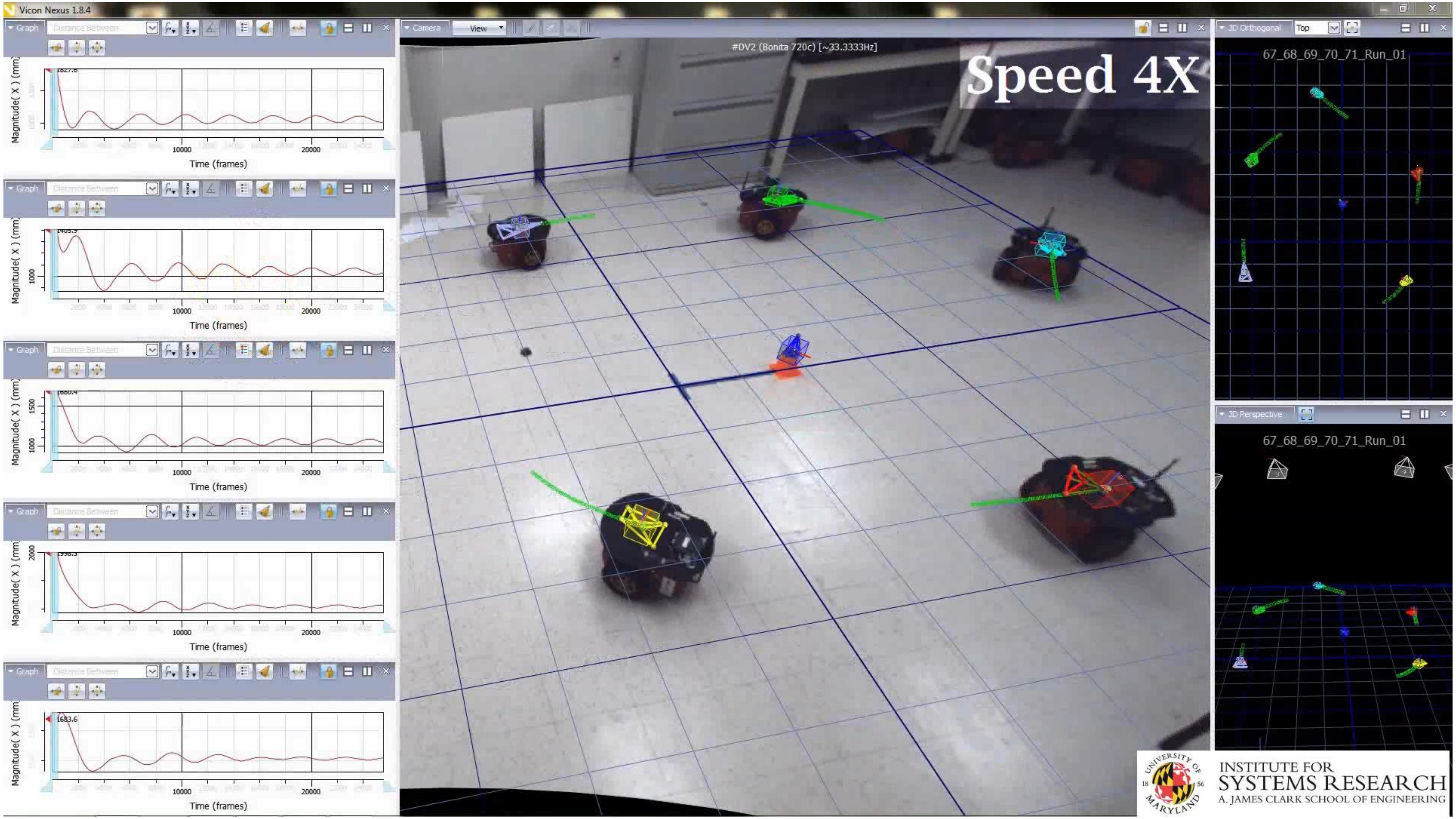


Implementation on Mobile Robots (2 robots)

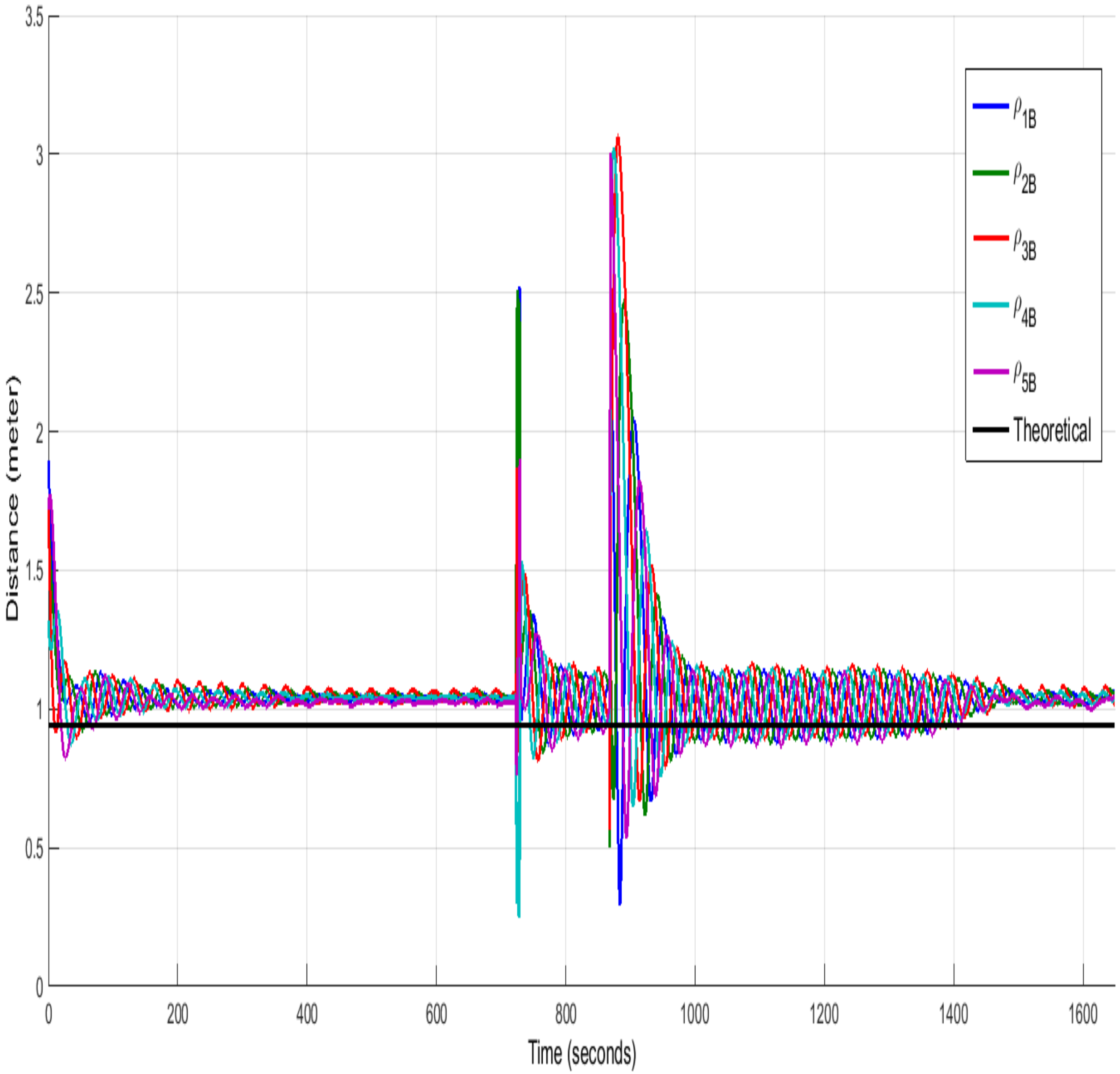


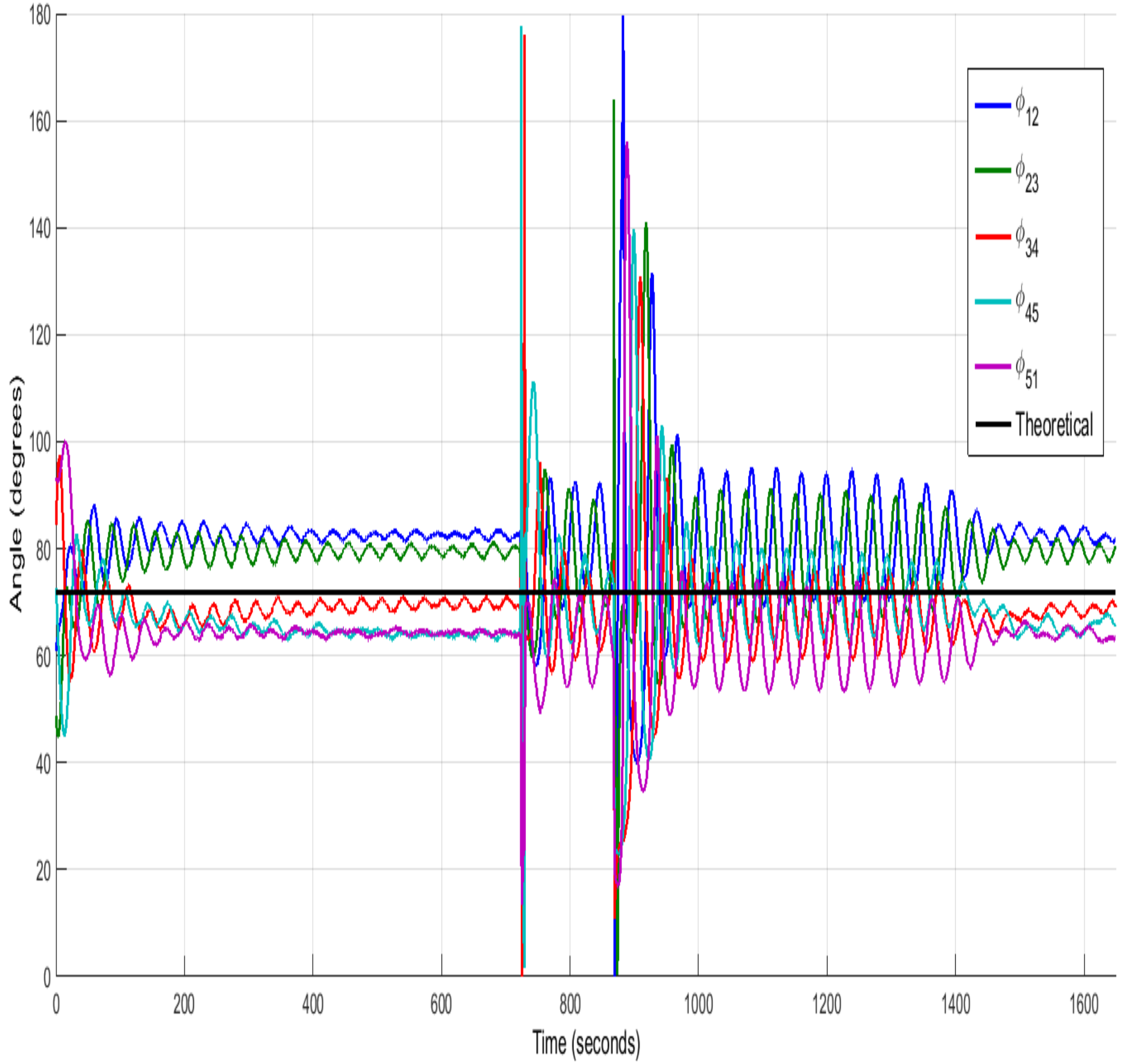


Implementation on Mobile Robots (5 robots)



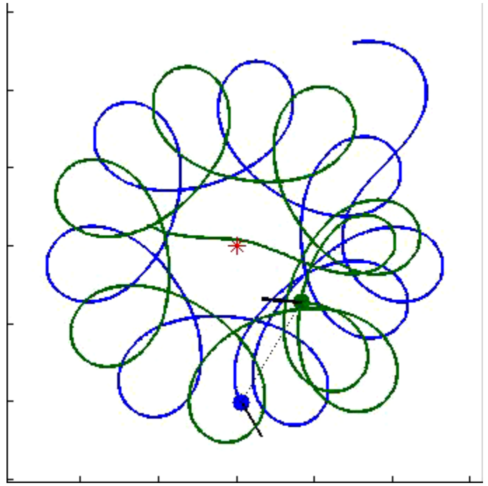
Implementation on Mobile Robots (5 robots)

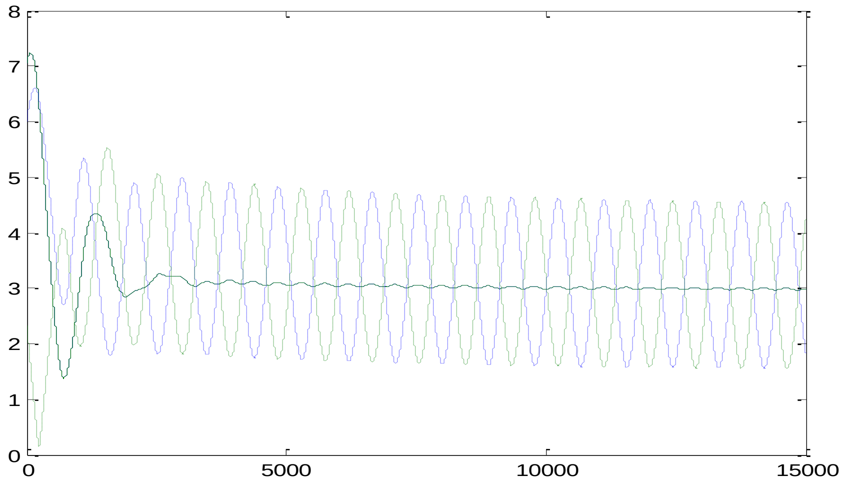




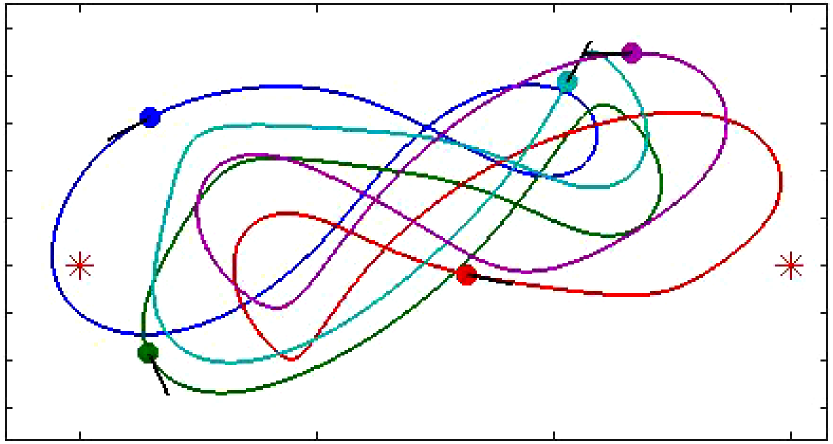
Future Work

- Invariant Submanifold/Conserved Quantity





- Multiple Beacon



- Explore-Exploit Trade-off

Acknowledgements

- Supported by:
    - AFOSR (including DURIP grant for mobile robot testbed)
    - ARL/ARO
    - ONR
    - USNA

  Thanks to:
    - P. S. Krishnaprasad, E. W. Justh, U. Halder

- Extended 8-page version of paper available on **arXiv**