PRINCETON UNIVERSITY

Investigating Group Behavior in Dance: An Evolutionary Dynamics Approach

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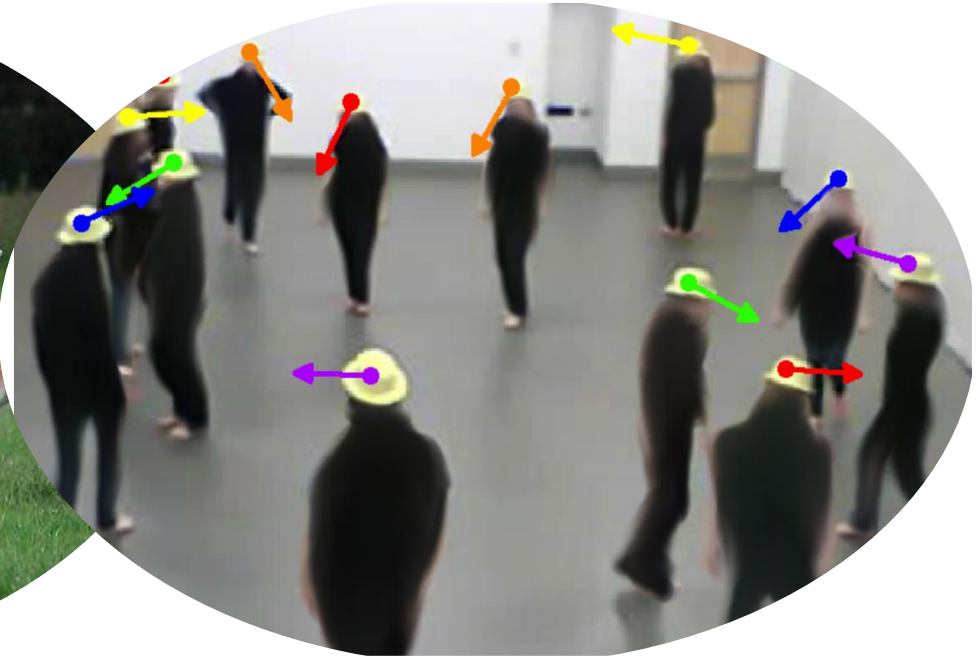
Social Decision Making in Animal & Human Groups

Nest Site Selection

Foraging Behavior

Flock Logic



Decision making as an Individual or Group

Motivation

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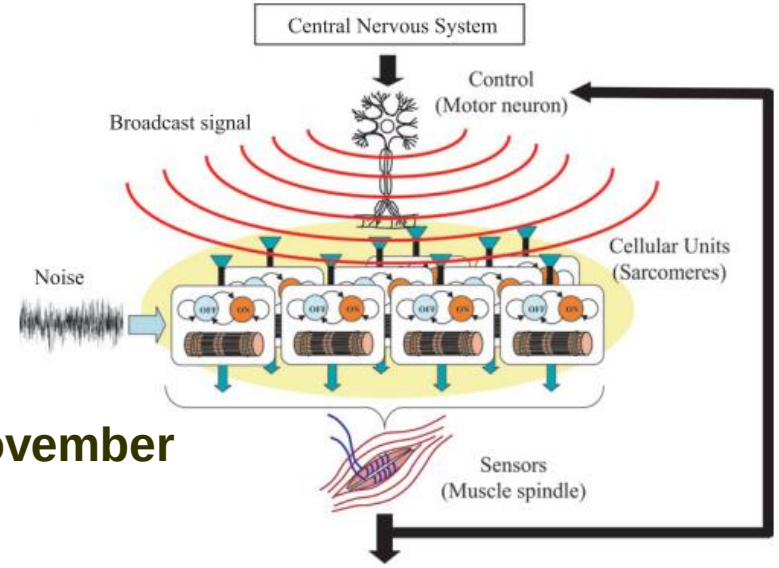
Broadcast Feedback of Stochastic Cellular Actuators Inspired by Biological Muscle Control



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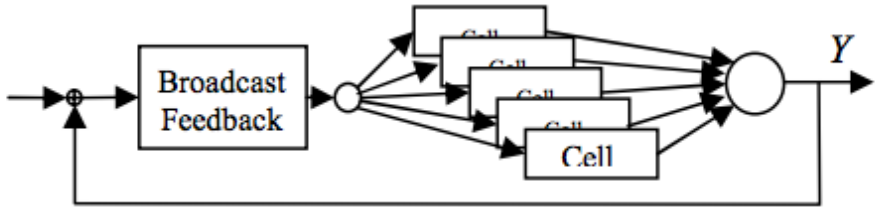
2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

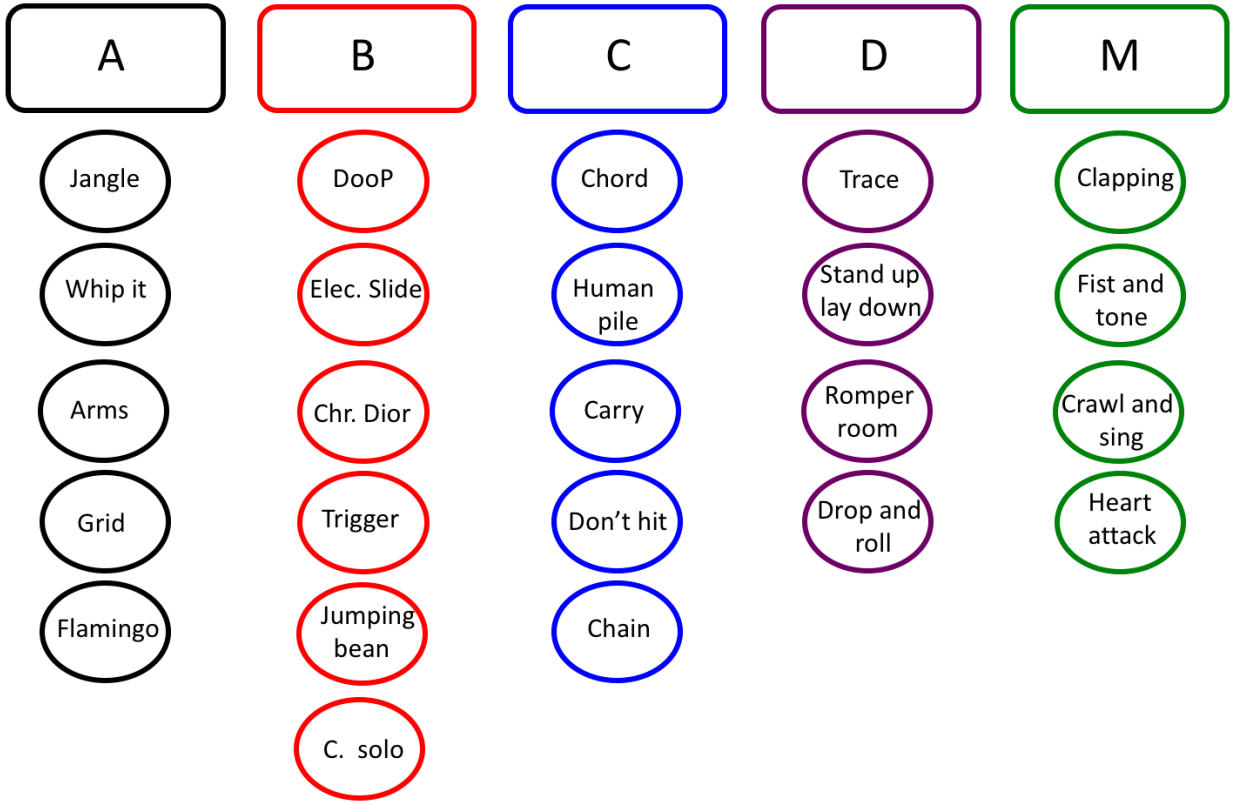
ThA10.2

Broadcast Feedback Control of Cell Populations Using Stochastic Lyapunov Functions with Application to Angiogenesis Regulation

Levi B. Wood, Anusuya Das, and H. Harry Asada, *Fellow, ASME*



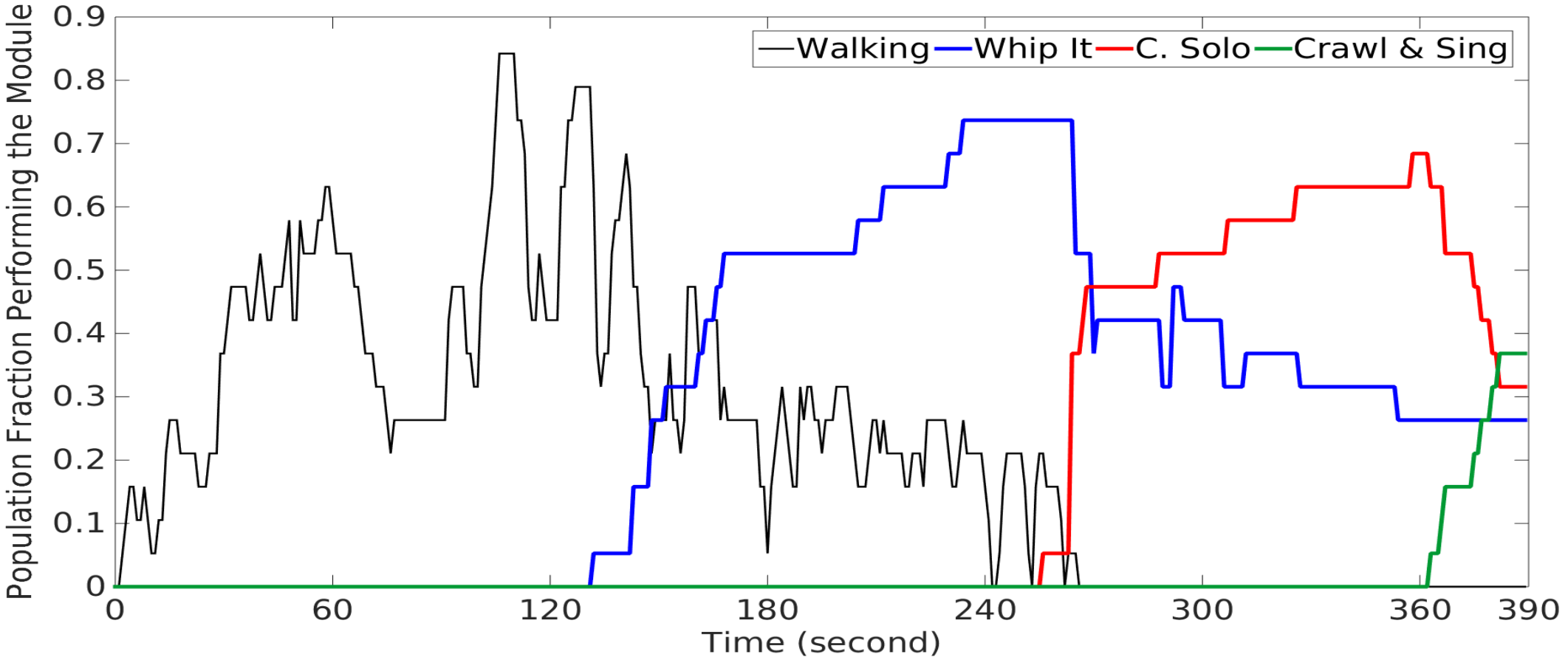
A Group Dance Performance: *There Might be Others*



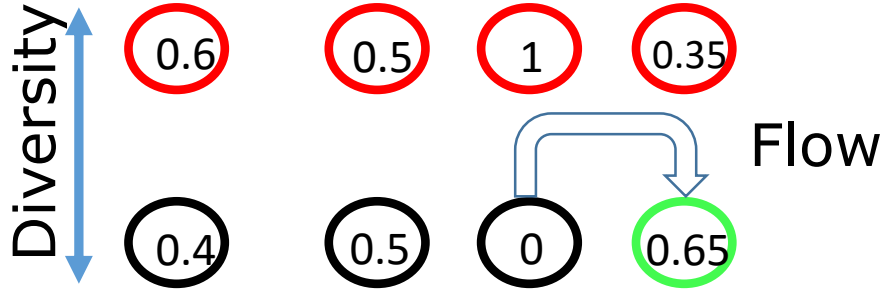
- Dancers choose the order of the modules.
- Maximum 2/3 modules are allowed simultaneously on stage.
- No leadership assignment (any dancer can introduce a new module).

A Group Dance Performance: *There Might be Others*

There Might be Others



Dancers Choose

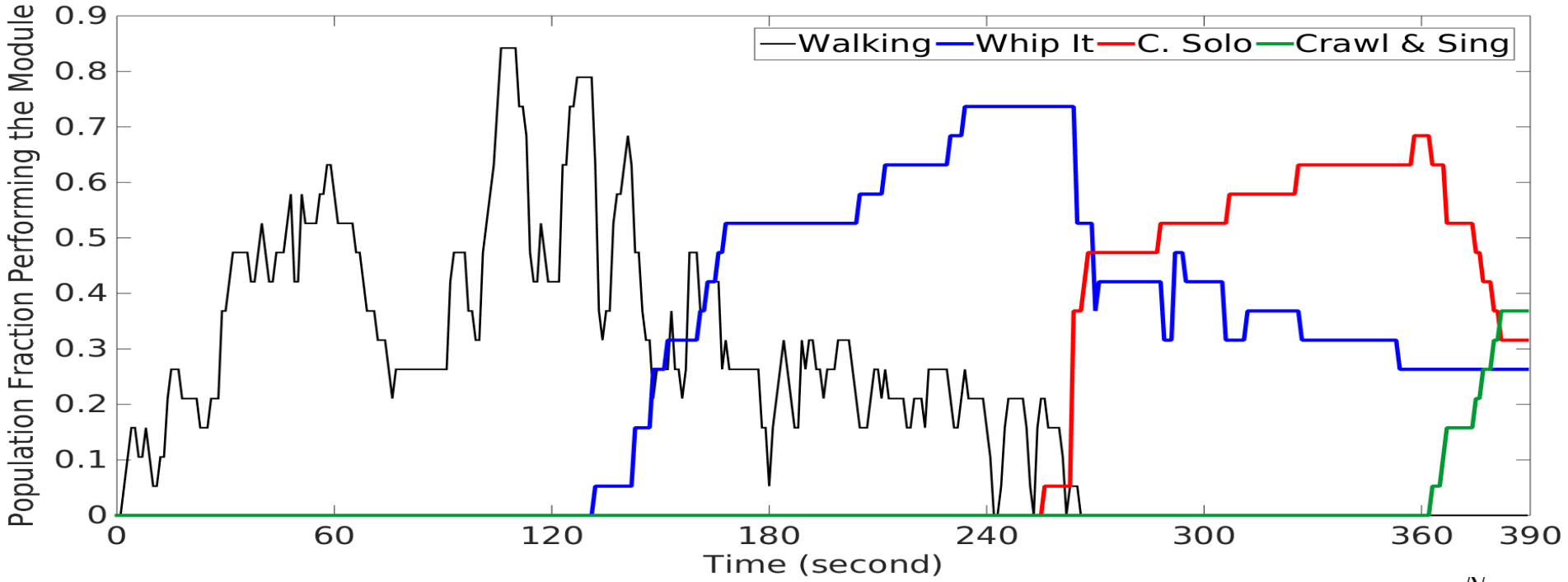


Diversity → Exploitation

vs.

Flow → Exploration

Evolutionary Dynamics to Study Group Behavior



Replicator Mutator Model: $\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i$

Fitness : $f_i = \sum_{j=1}^{N} b_{ji} x_j$

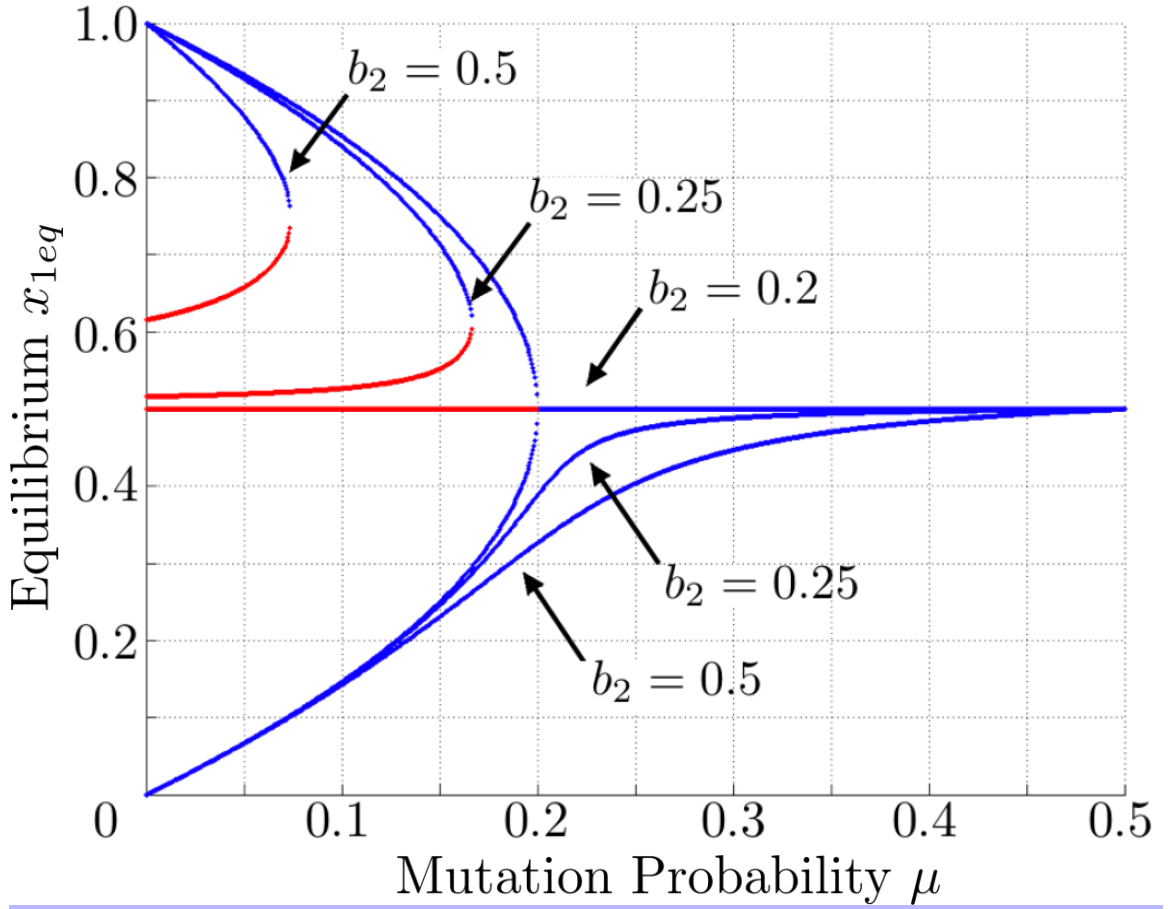
Average Fitness : $\phi = \sum_{i=1}^{N} f_i x_i$

Mutation : $\begin{cases} q_{ij} = \dfrac{\mu b_{ij}}{\sum_{j \neq i} b_{ij}} \\ q_{ii} = 1 - \mu \end{cases}$

K. M. Page and M. A. Nowak, "*Unifying evolutionary dynamics*", Journal of Theoretical Biology, 219(1):93–98, 2002.

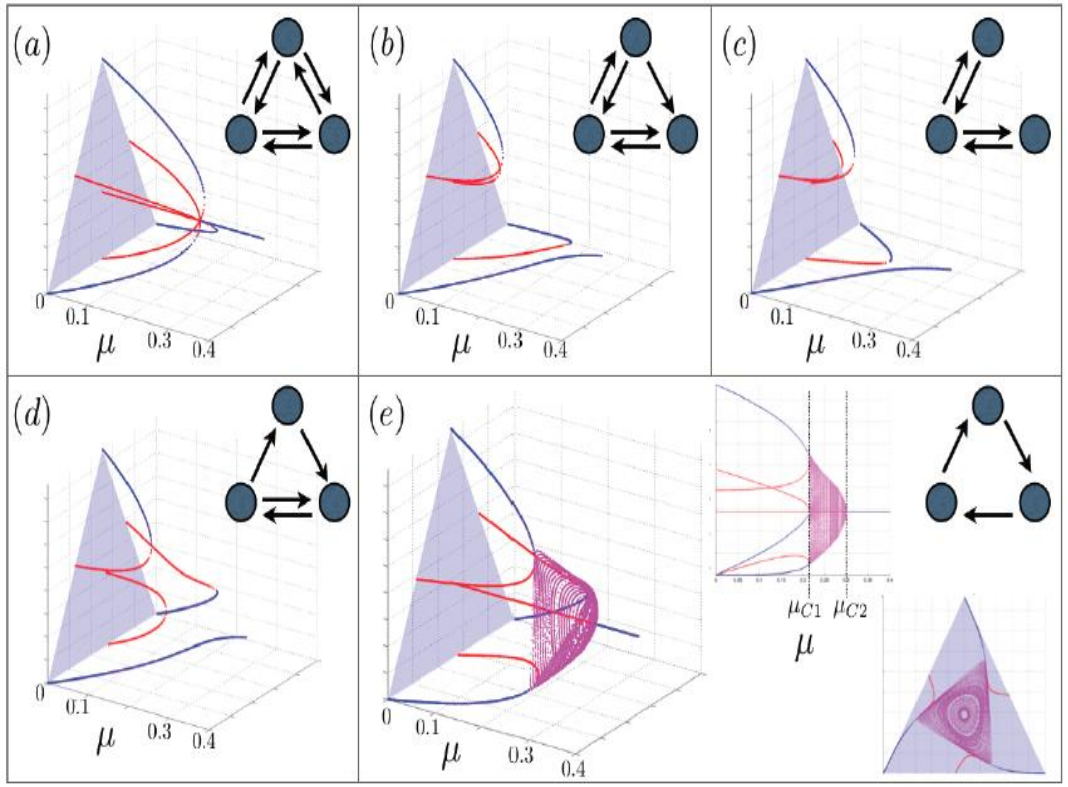
Previous Studies

Analysis for Two Strategies :



Komarova, Natalia L., and Simon A. Levin. "Eavesdropping and language dynamics." *Journal of theoretical biology* 264.1 (2010): 104-118.

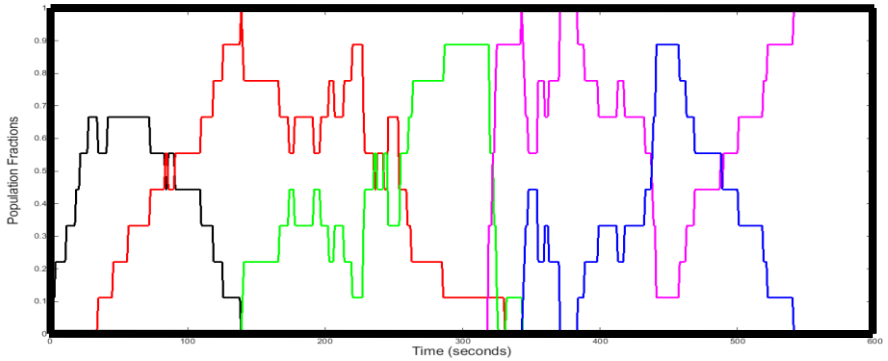
Analysis for Three Strategies :



Pais, Darren, Carlos H. Caicedo-Núnez, and Naomi E. Leonard. "Hopf bifurcations and limit cycles in evolutionary network dynamics." *SIAM Journal on Applied Dynamical Systems* 11.4 (2012): 1754-1784.

Evolutionary Dynamics to Study Group Behavior

Model modification motivated by the observations



$$\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i$$

Understanding the social decision making dynamics driven by artistic Explore-Exploit tension

Interpretation of Model for Dance Group Behavior

Perceived Dominance:

$$\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i$$

$$\dot{\eta}_i = K\big(S(x_i) - \alpha\big)$$

$$w_i = \frac{1}{1 + e^{-\eta_i}}, \qquad i \in 1, 2, \ldots, N$$

$\Longleftrightarrow$

$$\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i$$

$$\dot{w}_i = K\big(S(x_i) - \alpha\big) w_i (1 - w_i)$$

Perceived Dominance Threshold : $\alpha \in [0, 1]$

Awareness of Dominance for Two Strategies

Replicator Mutator Dynamics:

$$\dot{x}_1 = x_1\big(b + (1-b)x_1\big)(1-\mu-x_1) + (1-x_1)\big(1+(b-1)x_1\big)(\mu - x_1)$$

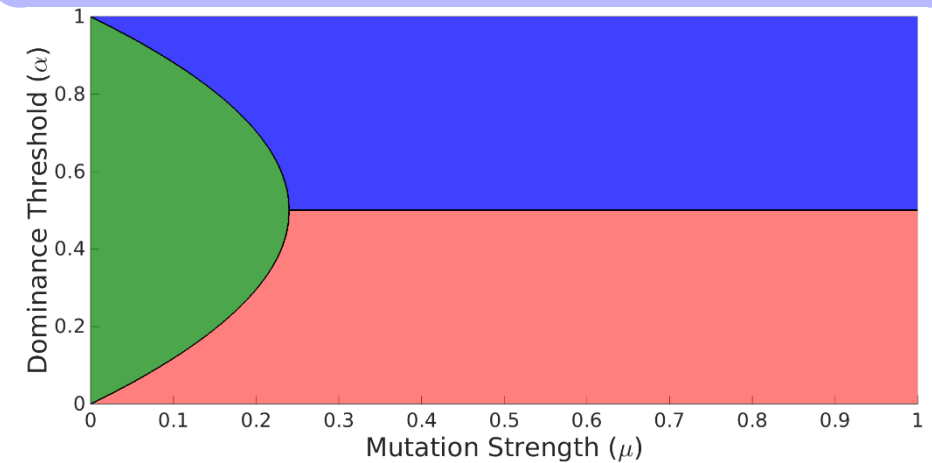
Awareness of Dominance:

$$\dot{w}_i = K\big(x_i - \alpha\big)w_i(1-w_i), \qquad i = 1,2 \qquad K > 0$$

- $\mu \geq (1-b)/4, \qquad \lim_{t\to\infty} w_i(t) = \begin{cases} 0 & \text{if } \alpha \in (0.5, 1] \\ 1 & \text{if } \alpha \in [0, 0.5) \end{cases}$.

- $\mu < (1-b)/4, \qquad \lim_{t\to\infty} w_i(t) = \begin{cases} 0 & \text{if } \alpha \in (v_u, 1] \\ 0 & \text{if } \alpha \in (v_l, v_u) \text{ and } x_i(0) < 0.5 \\ 1 & \text{if } \alpha \in (v_l, v_u) \text{ and } x_i(0) > 0.5 \\ 1 & \text{if } \alpha \in [0, v_l) \end{cases}$

where $v_u = 0.5 + \sqrt{0.25 - \mu/(1-b)}$ and $v_l = 0.5 - \sqrt{0.25 - \mu/(1-b)}$.

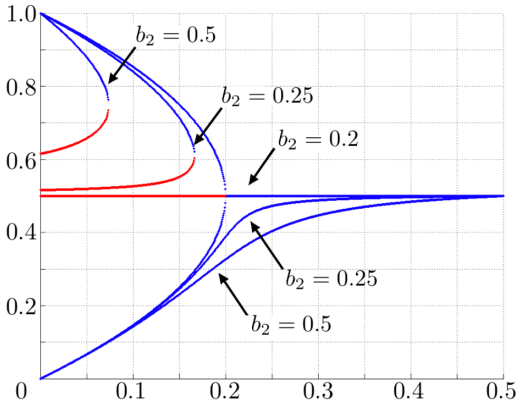


$w_i \to 0$

$w_i \to 1$

$w_i \to 1, \; w_j \to 0, \quad \text{if } x_i(0) > x_j(0)$

Loop Closure-Feedback Controlled Bifurcation



$$\dot{x}_i = \sum_{j=1}^{N} x_j f_j q_{ji} - \phi x_i$$

$\{x_i\}_{i=1}^{N}$

$$\mu = 1 - \max_{i \in \{1,2,\ldots,N\}} (w_i)$$

$$\dot{w}_i = K(x_i - \alpha) w_i (1 - w_i)$$

$\{w_i\}_{i=1}^{N}$

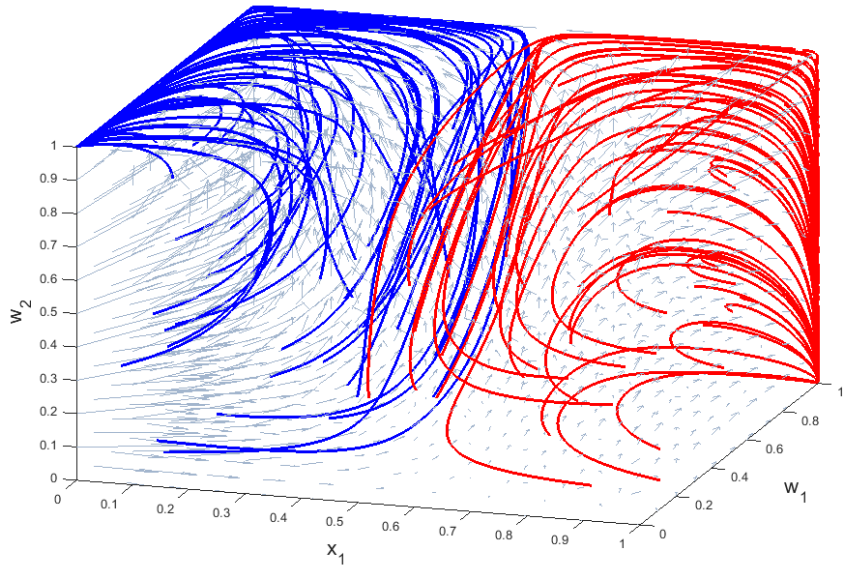
Closed Loop Behavior (Phase Portraits)

Forward Dynamics:

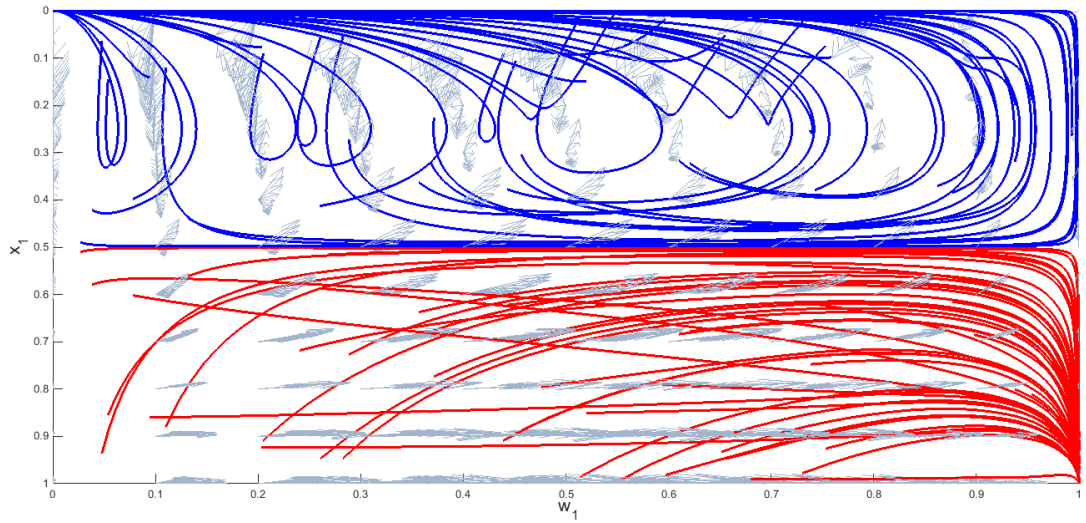
$$\dot{x}_1 = x_1\big(b + (1-b)x_1\big)\big(\max(w_1, w_2) - x_1\big) + (1 - x_1)\big(1 + (b-1)x_1\big)\big(1 - \max(w_1, w_2) - x_1\big)$$

Feedback Dynamics:

$$\dot{w}_i = K\big(x_i - \alpha\big)w_i(1 - w_i), \qquad i = 1, 2 \qquad K > 0$$



- $\alpha = 0.25$
- $b = 0.04$
- $K = 2$



Loop Closure for $K>>1$: Time-scale Separation

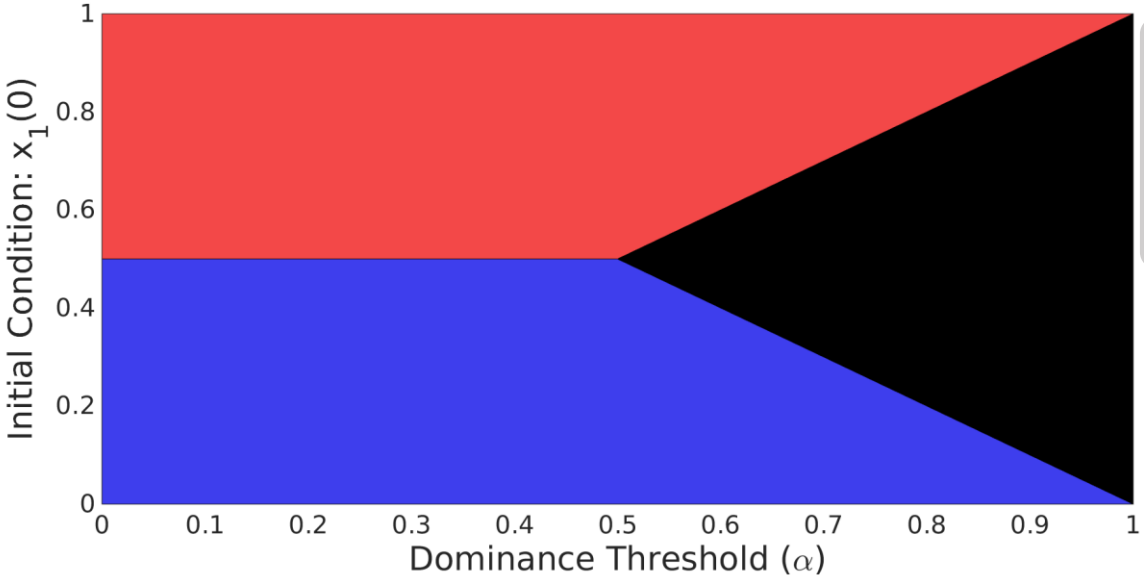
Slow Dynamics: Replicator-Mutator

$$\dot{x}_1 = x_1\big(b+(1-b)x_1\big)\big(\max(w_1,w_2)-x_1\big) + (1-x_1)\big(1+(b-1)x_1\big)\big(1-\max(w_1,w_2)-x_1\big)$$

Fast Dynamics: Awareness of Dominance

$$\epsilon\dot{w}_i = \big(x_i-\alpha\big)w_i(1-w_i), \qquad \epsilon \triangleq \frac{1}{K} \ll 1$$

$$\lim_{t\to\infty} x_1(t) = \begin{cases} 1 & x_1(0) > \max(\alpha, 0.5) \\ 0.5 & \alpha > x_1(0) > 1-\alpha \\ 0 & x_1(0) < \min(1-\alpha, 0.5) \end{cases}$$



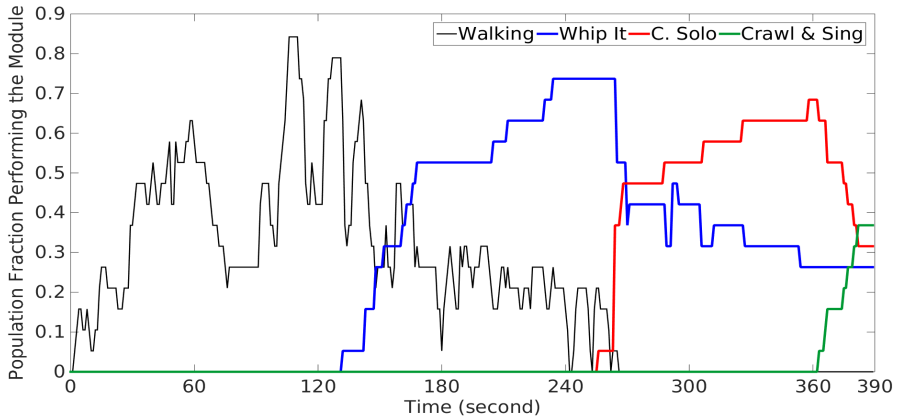
Let, $x_i(0) \geq x_j(0)$, where $i, j \in \{1,2\}$ and $i \neq j$.
Then, $\lim_{t\to+\infty} x_i(t) \geq \lim_{t\to+\infty} x_j(t)$.

$x_1 \to 1, \quad x_2 \to 0$

$x_1 \to 0, \quad x_2 \to 1$

$x_1 \to 0.5, \quad x_2 \to 0.5$

Conclusion and Future Work



Diversity → Exploitation

vs.

Flow → Exploration

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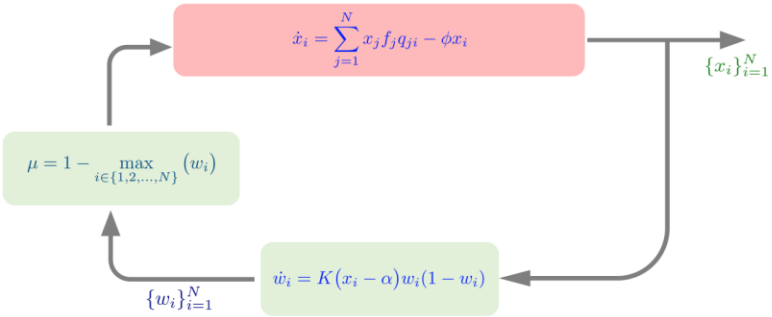
$$\dot{w}_i = K\big(S(x_i) - \alpha\big) w_i (1 - w_i)$$



Conditions for Exploration Phase → Exploitation?

Thank You