DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Energy-Based Control of a Flexible Beam

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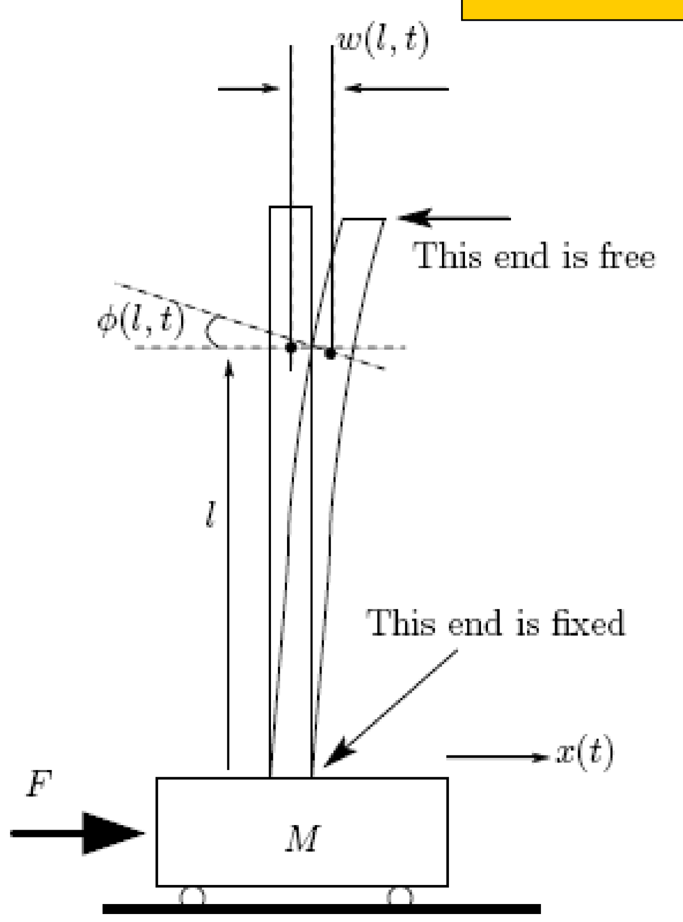
Introduction



•In this work we are interested to stabilize the flexible beam under disturbances.

•The control effort is the horizontal motion of the cart.

•The system is modeled in a port-controlled Hamiltonian (PCH) framework.

- Both the beam and the cart are mechanical energy storing elements. They exchange energy between themselves and the control objective has been achieved by regulating this process.
- We adopt the *energy Casimir method* to achieve this goal.

Modeling

•Hamiltonian (total energy) for the flexible beam ($\mathcal{H}_{\mathcal{B}}$):

$$\frac{1}{2}\int_0^L \left[\rho\left(\frac{\partial w}{\partial t}+\dot{x}\right)^2 + I_\rho\left(\frac{\partial \phi}{\partial t}\right)^2 + K\left(\phi-\frac{\partial w}{\partial l}\right)^2 + EI\left(\frac{\partial \phi}{\partial l}\right)^2\right] dl + \frac{\rho L^2 g}{2}$$

•To model this *infinite-dimensional system* we need to introduce the following *differential 1-forms*:

- *Translational Momentum*: $p_t(l,t) \triangleq \rho\left(\frac{\partial z}{\partial t}\right) dl$
- *Rotational Momentum*: $p_r(l,t) \triangleq I_\rho\left(\frac{\partial \phi}{\partial t}\right) dl$
- *Translational Strain*: $\epsilon_t(l,t) \triangleq \left(\frac{\partial z}{\partial l}-\phi\right) dl$
- *Rotational Strain*: $\epsilon_r(l,t) \triangleq \left(\frac{\partial \phi}{\partial l}\right) dl$

where, $z(l,t) \triangleq w(l,t) + x(t) \quad \forall t$

•Now we'll define a *Dirac structure* ($\mathbb{D}$), essentially a notion of *orthogonality* to express power conservation in the system, over the space of *flows* and *efforts* of the system.

•Once the Dirac structure has been defined the port-Hamiltonian formulation will automatically follow.

•Exploiting the underlying Dirac structure, the *infinite dimensional port Hamiltonian model* for the flexible beam is obtained as:

$$\frac{\partial}{\partial t}\begin{bmatrix} p_t \\ p_r \\ \epsilon_t \\ \epsilon_r \end{bmatrix} = \begin{bmatrix} 0 & 0 & d & 0 \\ 0 & 0 & * & d \\ d & -* & 0 & 0 \\ 0 & d & 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_{p_t}\mathcal{H}_{\mathcal{B}} \\ \delta_{p_r}\mathcal{H}_{\mathcal{B}} \\ \delta_{\epsilon_t}\mathcal{H}_{\mathcal{B}} \\ \delta_{\epsilon_r}\mathcal{H}_{\mathcal{B}} \end{bmatrix} \text{ and, } \begin{bmatrix} f_b^t \\ f_b^r \\ e_b^t \\ e_b^r \end{bmatrix} = \begin{bmatrix} \delta_{p_t}\mathcal{H}_{\mathcal{B}}|_{\partial\mathcal{D}} \\ \delta_{p_r}\mathcal{H}_{\mathcal{B}}|_{\partial\mathcal{D}} \\ \delta_{\epsilon_t}\mathcal{H}_{\mathcal{B}}|_{\partial\mathcal{D}} \\ \delta_{\epsilon_r}\mathcal{H}_{\mathcal{B}}|_{\partial\mathcal{D}} \end{bmatrix}$$

Hodge Star Operator ($*$)

Exterior Derivative (d)

•Cart: a typical second order system governed by a single configuration variable (the displacement x)

•Controller: one configuration variable ($\tilde{x}$)

•The controller and the cart is considered as an integrated system.

Motivation:

Separation of the finite dimensional and infinite dimensional parts of the overall system

•Combined Hamiltonian: $H_c = H_{cart} + H_{controller}$

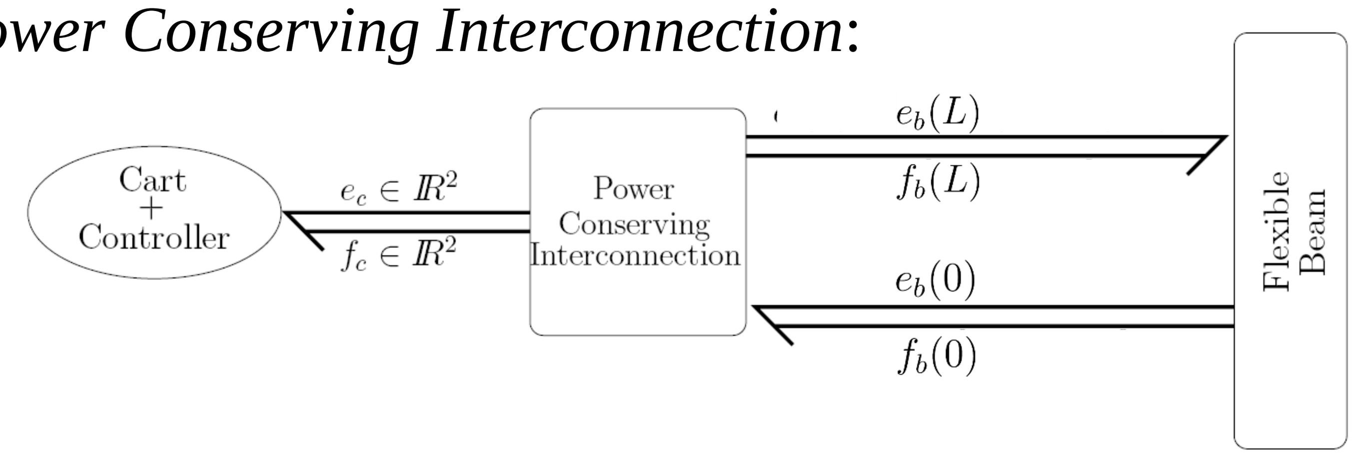
•*Dynamics of integrated system*:

$$\begin{bmatrix} \dot{q}_c \\ \dot{p}_c \end{bmatrix} = \left(\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & D_c \end{bmatrix}\right)\begin{bmatrix} \frac{\partial H_c}{\partial q_c} \\ \frac{\partial H_c}{\partial p_c} \end{bmatrix} + \begin{bmatrix} 0 \\ G_c \end{bmatrix} f_c$$

and, $e_c = G_c^T \partial_{p_c} H_c$

D_c >0 $\qquad f_{c2} = 0$

•*Power Conserving Interconnection*:



Stabilization

•Equilibrium Configuration: Upright position of the beam with stationary cart.

•Energy-Casimir approach is adopted to stabilize the *relative equilibrium.*

•Casimir functionals are invariant of the trajectories of the system and relates the states of the controller with those of the cart and the beam.

•Number of Casimir functional for this system: 2

•Casimir Functional:

$$\mathcal{C}_i = q_{c_i} + \int_{\mathcal{D}} \left[(k_3^i + k_1^i l)p_t + k_1^i p_r + k_2^i \epsilon_t + (k_4^i - k_2^i l)\epsilon_r\right]$$

•By suitable choice for e_c, we get: $G_c = \left(\frac{\partial \mathcal{C}}{\partial q_c}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

•H_c will be chosen in such a way that the Hamiltonian for the closed loop system will be *invariant* under the translation of the cart and the controller variable.

$$H_c = \frac{1}{2}{p_c}^T {M_c}^{-1} p_c + \frac{1}{2}K_{c1}(\tilde{\mathcal{C}}_1)^2 + \frac{1}{2}K_{c2}(\tilde{\mathcal{C}}_2)^2$$

•The stability has been proved using the *convexity condition*.

•The presence of *dissipation* in the finite dimensional part of the system guarantees the asymptotic stability.

•Controller Structure:

•*Controller Hamiltonian*:

$$H_{controller} = \frac{1}{2}\tilde{M}\dot{\tilde{x}}^2 + \frac{1}{2}K_{c2}(\tilde{x}-\mathcal{C}_2)^2 + \frac{1}{2}K_{c1}(x-\mathcal{C}_1)^2$$

•*Cart and Controller Dynamics*:

$$M\ddot{x} = F$$
$$\tilde{M}\ddot{\tilde{x}} = -K_{c2}(\tilde{x}-\mathcal{C}_2) - a_2\dot{x} - a_3\dot{\tilde{x}}$$

•*Feedback Control Law*:

$$F = -K_{c1}(x-\mathcal{C}_1) - a_1\dot{x} - a_2\dot{\tilde{x}} + f_{c1}$$

Discussion

•An important feature of this control methodology is that it is solution free; that is: the solution of the PDEs defining the system is not required to obtain a stabilizing controller.

•This theoretical formulation and extraction of control law can be applied to solve issues related to *stabilization of flexible structures*.

References:

•Biswadip Dey, "Stabilizing a Flexible Beam on a Cart: A Distributed Port Hamiltonian Approach", M. Tech Dissertation, IIT Bombay, 2008.

•Ravi N. Banavar and Biswadip Dey, "Stabilizing a Flexible Beam on a Cart: A Distributed Port Hamiltonian Approach" in *Proceedings of the European Control Conference (ECC)*, Budapest, Hungary, Aug. 2009.