MAE 544 - Nonlinear Control

Fall 2017

Homework # 1

- Assigned: September 29, 2017.
- Due: Thursday, October 5, 2017 by 3:00 pm in class.

1. *Let* v_1*,* v_2*,* v_3 *and* v_4 *be three linearly independent vectors in the Euclidean space* $\mathbb{R}^n$*. Can you obtain a set of linearly independent vectors* w_1*,* w_2*,* w_3 *and* w_4 *such that the following hold true:* $span(\{v_1, v_2, v_3, v_4\}) = span(\{w_1, w_2, w_3, w_4\})$*;* $w_1 = v_1$*; and* $w_i^T w_j = 1$ *if and only if* $i = j$ *and zero otherwise. Explain your answer.*

2. *Let* $L(\mathbb{R}^n, \mathbb{R})$ *be the set of real-valued Lipschitz continuous functions on* $\mathbb{R}^n$*. Prove that* $L(\mathbb{R}^n, \mathbb{R})$ *is a vector space with the following operations:*
   - *Vector Addition:* $\Big(f + g\Big)(x) = f(x) + g(x)$ *for any* $f, g \in L(\mathbb{R}^n, \mathbb{R})$*.*
   - *Scalar Multiplication Addition:* $\Big(\alpha f\Big)(x) = \alpha f(x)$ *for any* $\alpha \in \mathbb{R}$ *and* $f \in L(\mathbb{R}^n, \mathbb{R})$*.*

3. *Consider the 2-Sphere* $\mathcal{S}^2 \subset \mathbb{R}^3$ *defined as*

$$\mathcal{S}^2 = \Big\{(x_1, x_2, x_3) \in \mathbb{R}^3 \Big| x_1^2 + x_2^2 + x_3^2 = 1\Big\}.$$

   (a) Construct an atlas for $\mathcal{S}^2$*.*
   (b) Compute the associated transition maps. Are they smooth?

4. *Let* $\mathcal{M}$ *and* $\mathcal{N}$ *be two smooth manifolds, and* $\Phi : \mathcal{M} \to \mathcal{N}$*. Then show that* Φ *is* $\mathcal{C}^\infty$ *(i.e. smooth) if and only if the composition* $f \circ \Phi$ *is* $\mathcal{C}^\infty$ *for every* $\mathcal{C}^\infty$ *function* $f : \mathcal{N} \to \mathbb{R}$*.*