MAE 544 - Nonlinear Control

Fall 2017

Homework # 3

- Assigned: November 2, 2017.
- Due: November 9, 2017 (Thursday) by 3:00 pm in class.

1. *A smooth vector field $X \in \mathfrak{X}(G)$ on a Lie-group G is called* right-invariant *if*

$$X(R_g h) = (R_g)_* X(h) \tag{1}$$

for every $g, h \in G$. Let $\mathfrak{X}_R(G) \subset \mathfrak{X}(G)$ be the space of right-invariant vector fields on G. Show that $\mathfrak{X}_R(G)$ is isomorphic to $\mathfrak{X}_L(G)$, i.e. the space of left-invariant vector fields on G.

2. *Let $\{X_i\}_{i=1}^N$ be a set of linearly independent vector fields on a manifold $\mathcal{M}$, and for any $i, j \in \{1, \ldots, N\}$ there are smooth functions $\gamma_{ijk} \in \mathcal{C}^\infty(\mathcal{M})$ such that*

$$[X_i, X_j] = \sum_k \gamma_{ijk} X_k. \tag{2}$$

Show that the span of these vector fields defines an integrable distribution.

3. *Let $\mathcal{M} = \mathbb{R}^3$, and define the distribution $\mathcal{D}$ on $\mathcal{M}$ as the span of the vector fields given by*

$$\begin{aligned} F_1 &= 2x_2 \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \\ F_2 &= \frac{\partial}{\partial x_1} + x_2 \frac{\partial}{\partial x_3}. \end{aligned} \tag{3}$$

Is $\mathcal{D}$ non-singular? Is it involutive?

4. *Let $\mathcal{M} = \mathbb{R}^3$, and define the distribution $\mathcal{D}$ on $\mathcal{M}$ as the span of the vector fields given by*

$$\begin{aligned} F_1 &= x_1 \frac{\partial}{\partial x_1} + (1 + x_3) \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3} \\ F_2 &= x_1^2 \frac{\partial}{\partial x_1} + x_1^2 \frac{\partial}{\partial x_2} \\ F_3 &= x_1 x_2 \frac{\partial}{\partial x_1} + (x_2 x_3 + x_2) \frac{\partial}{\partial x_2} + x_2 \frac{\partial}{\partial x_3} \end{aligned} \tag{4}$$

Is $\mathcal{D}$ non-singular? Is it involutive?

5. *Consider the following kinematic model of a vertically upright coin rolling on the $x - y$ plane*

$$\dot{x} = u_1 \cos\theta, \quad \dot{y} = u_1 \sin\theta, \quad \dot{\phi} = k u_1, \quad \dot{\theta} = u_2, \tag{5}$$

where $k > 0$. Is (5) controllable? Is it small time locally controllable (STLC) even when the controls are restricted to be positive, i.e. $u_1 \in (0, \infty)$ and $u_2 \in (0, \infty)$?