MAE 544 - Nonlinear Control

Fall 2017

Homework # 4

- Assigned: November 30, 2017.
- Due: December 7, 2017 (Thursday) by 3:00 pm in class.

1. Consider the system:

$$\dot{x}_1 = -x_2 + \epsilon x_1(x_1^2 + x_2^2)\sin(x_1^2 + x_2^2)$$
$$\dot{x}_2 = x_1 + \epsilon x_2(x_1^2 + x_2^2)\sin(x_1^2 + x_2^2)$$

   where $\epsilon \in [-1, 1]$. Can you use linearization to show stability of the origin $(0, 0)$? If not, then use the direct method of Lyapunov to investigate stability in this case.

2. As we know, the linear time invariant system $\dot{x}(t) = Ax(t)$, $t \geq 0$, is asymptotically stable if the matrix A is Hurwitz. Can we say that a time varying system $\dot{x}(t) = A(t)x(t)$, $t \geq 0$ is asymptotically stable if $A(t)$ is Hurwitz for all $t \geq 0$?
   In addition, show that the system $\dot{x}(t) = A(t)x(t)$ is asymptotically stable if $A(t) + A^T(t)$ is a Hurwitz matrix for all $t \geq 0$.

3. Consider the nonlinear system:

$$x = Ax + f(x) + B\text{sat}(u),$$

   where $f : \mathbb{R}^n \to \mathbb{R}^n$ is globally Lipschitz with Lipschitz constant $\Gamma_f > 0$ and $f(0) = 0$. Let $P = P^T > 0$ be a solution for the Lyapunov equation

$$A^T P + PA = -Q,$$

   where $Q = Q^T > 0$. Show that the origin ($x = 0$) is globally stabilizable by a linear state feedback $u = Kx$ if

$$\Gamma_f < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}.$$

   Also, design a feedback law that stabilizes the system.

4. Show that if $f \in L_1 \cap L_\infty$, then $f \in L_p$ for every $p \in [1, \infty)$.

5. Consider the system:

$$\dot{y} = -2y + \text{sat}(y) + u, \qquad y(0) = y_0.$$

   Show that the system is passive. Is the system strictly passive?