MAE 544 - Nonlinear Control

Fall 2017

Homework # 5

- Assigned: December 7, 2017.
- Due: December 14, 2017 (Thursday). *Please place it in my mailbox in the MAE mailroom or email me the scanned copy.*

1. Consider the system:

$$\dot{x}_1 = x_1^2 - x_1^3 + x_2$$
$$\dot{x}_2 = u.$$

Using back-stepping design a controller which stabilizes the origin globally.

2. Consider the system:

$$\dot{x}_1 = x_2 - x_3$$
$$\dot{x}_2 = -x_1 x_3 - x_2 + u$$
$$\dot{x}_3 = -x_1 + u.$$

Does there exist an output function $y = h(x)$ for which the relative degree of this system is 3? If possible, design a feedback linearizing control law $u : \mathbb{R}^3 \to \mathbb{R}$ such that the closed-loop linear dynamics have eigenvalues at $\{-1, -2 \pm j\}$.

3. The dynamics of a single link manipulator with flexible joint can be expressed as

$$\ddot{q}_1 - (q_2 - q_1) = u$$
$$\ddot{q}_2 + 10 \sin q_2 + (q_2 - q_1) = 0.$$

By defining the state vector as $x = [q_1, \dot{q}_1, q_2, \dot{q}_2]$, represent the dynamics in a state-space form. Then consider the single-input single-output control problems with either $y = q_1$ or $y = q_2$ as the output. In both of these cases, perform feedback linearization, and compute and simulate the zero dynamics (using Matlab or Python). Which output would you prefer to control? Please explain your answer.