Lecture 24.

MAE 544

NONLINEAR CONTROL.

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BD| ①

ZERO DYNAMICS:—

Consider the system

$$\boxed{\begin{aligned}\dot{x} &= f(x) + u g(x)\\ y &= h(x)\end{aligned}} \quad \text{——— (1)}$$

where $x \in \mathbb{R}^n$; $y, u \in \mathbb{R}$; $f, g \in \mathfrak{X}(\mathbb{R}^n)$ and $h \in C^\infty(\mathbb{R}^n)$ and assume it has a relative degree of γ. Then a control law —

$$\boxed{\bar{u}(x) = -\frac{L_f^{\gamma} h(x)}{L_g L_f^{\gamma-1} h(x)}} \quad \text{——— (2)}$$

will keep M defined as —

$$\boxed{M \triangleq \left\{ x \,\middle|\, L_f^{k} h(x) = 0,\ 0 \leq k \leq \gamma - 1 \right\}} \quad \text{——— (3)}$$

invariant under the dynamics. In (ξ, η) coordinates the dynamics (1) on M will look like —

$$\xi = 0$$

$$\boxed{\dot{\eta} = q(0, \eta)} \quad \text{——— (4)}$$

(4) is called the zero dynamics associated with (1). In a linear system context, the eigenvalues of the linear dynamics of η correspond to the open-loop zeros. We used to call a linear system minimum phase if its zero were in the open left half plane. From this perspective minimum phase was equivalent to

Stability of the zero dynamics.

Now we assume x_0 is an equilibrium of (1), such that, $h(x_0)=0$ as well. Then $\xi=0$ at x_0, and we can always make $\eta=0$ at this point. Thus $(\xi,\eta)=(0,0)$ is an equilibrium for the system in normal coordinates, and therefore $L_f^{r} h(x_0)=0$ and $q(0,0)=0$.

The original system (1) is locally asymptotically (resp. exponentially) minimum phase at x_0 if $\eta=0$ is an asymptotically (resp. exponentially) stable equilibrium of $\dot{\eta}=q(0,\eta)$ dynamics.

If $\left.\frac{\partial q}{\partial \eta}\right|_{\eta=0}$ has eigenvalues in the open left half plane, then (1) is locally exponentially minimum phase, and if it has some eigenvalues on the open right half plane, then (1) is non-minimum phase.

TRACKING:

Now we focus on finding an input and ~~input~~ initial condition for (1) so that its output $y(t)$ can exactly track a desired output $y_d(t)$, i.e. we want ξ to be —

$$\xi(t)=\xi^d(t)=\begin{pmatrix} y_d(t) \\ \dot{y}_d(t) \\ \vdots \\ y_d^{(r-1)}(t) \end{pmatrix}$$

Then, as we have —

$$y_d^{(\gamma)}(t) = \frac{d^\gamma y_d}{dt^\gamma}(t) = b(\xi^d, \eta) + u\, a(\xi^d, \eta),$$

the corresponding control input is given by —

$$\boxed{u_d = \frac{1}{a(\xi^d, \eta)}\left[\frac{d^\gamma y_d}{dt^\gamma} - b(\xi^d, \eta)\right]}.$$

Also, we need, $\xi(0) = \xi^d(0)$. The corresponding η-dynamics is given by —

$$\dot{\eta} = q(\xi^d, \eta) \longleftarrow$$ There will be complication if the system is non-minimum phase.

with any arbitrary $\eta(0)$. However there are some challenges with this approach —

(i) We introduce differentiation in the controller which makes the system more susceptible to noise.

(ii) We need an exact copy of the zero dynamics.

(iii) u_d is not guaranteed to be well defined if y_d and its first $(\gamma-1)$-derivatives are not small.

- Another alternative approach is to consider asymptotic tracking.

Let's define —

$$\boxed{u = \frac{1}{a(\xi, \eta)}\left[-b(\xi, \eta) + y_d^{(\gamma)} - \sum_{i=1}^{\gamma} c_{i-1}\left(\xi_i - y_d^{(i-1)}\right)\right]}$$

Then we can show that the evolution of tracking error will be governed by the c_i's. Hence, we can choose c_i's such that the error converges to zero.

LOCAL ASYMPTOTIC STABILIZATION:

Suppose (1) has a relative degree of γ and it locally exponentially minimum phase. Then, by using,

$$u = \frac{1}{L_g L_f^{\gamma-1} h(x)}\left[-L_f^{\gamma} h(x) + v\right],$$

the dynamic of ξ (the normal form coordinate can be expressed as—

$$\dot{\xi}_1 = \xi_2,\ \dot{\xi}_2 = \xi_3, \text{—}, \dot{\xi}_\gamma = v.$$

Then by choosing, $v = -\sum_{i=1}^{\gamma} \alpha_{i-1}\xi_i$

we can make the eigenvalues of ξ-dynamics the roots of $(s^\gamma + \alpha_{\gamma-1}s^{\gamma-1} + \text{--} + \alpha_1 s + \alpha_0)$. Thus by choosing α_i's appropriately we can defin u as—

$$u = \frac{-1}{L_g L_f^{\gamma-1} h(x)}\left[\alpha_0 h(x) + \alpha_1 L_f h(x) + \text{--} + \alpha_{\gamma-1} L_f^{\gamma-1} h(x) + L_f^{\gamma} h(x)\right]$$

which will make the closed loop system locally exponentially stable.

SLIDING MODE CONTROL:—

It originated as variable structure control (in 1950s). The key idea was to vary the system <u>structure</u> to get stabilization

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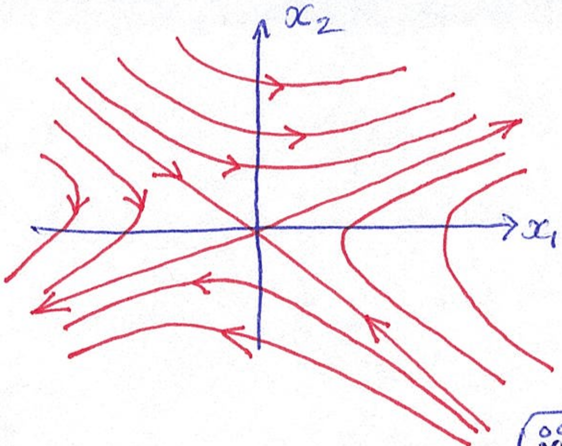
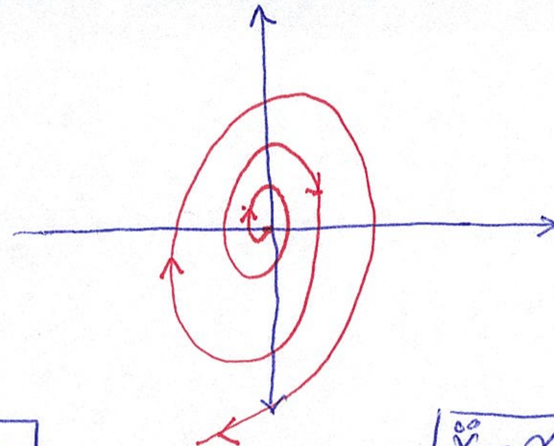
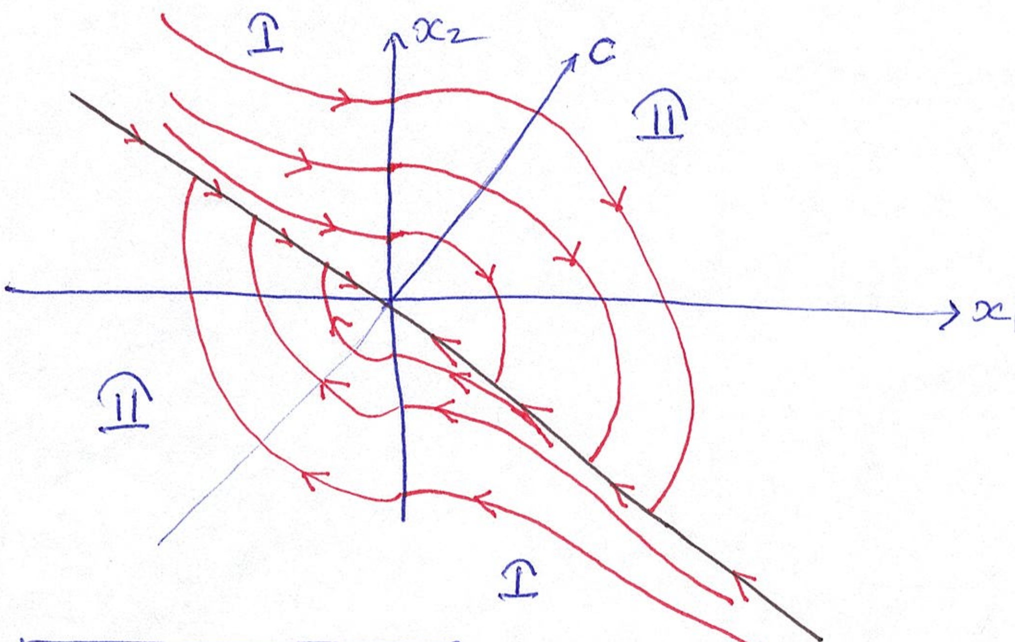




$$\boxed{\ddot{x} - \alpha\dot{x} - \beta x = 0}$$ I

$$\boxed{\ddot{x} - \alpha\dot{x} + \beta x = 0}$$ II



$$\boxed{\ddot{x} - \alpha\dot{x} + u = 0}$$

Choose $\boxed{u = \mathrm{sgn}\big(\psi(x)\big)\beta}$

where, $\psi(x) = x_1(c_1x_1 + c_2x_2)$.