Data Assimilation
Optimal Fitting, Cross-Validation, and Feedback Laws

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Workshop on *Geometry of Collective Behavior: Control, Dynamics and Reconstruction*
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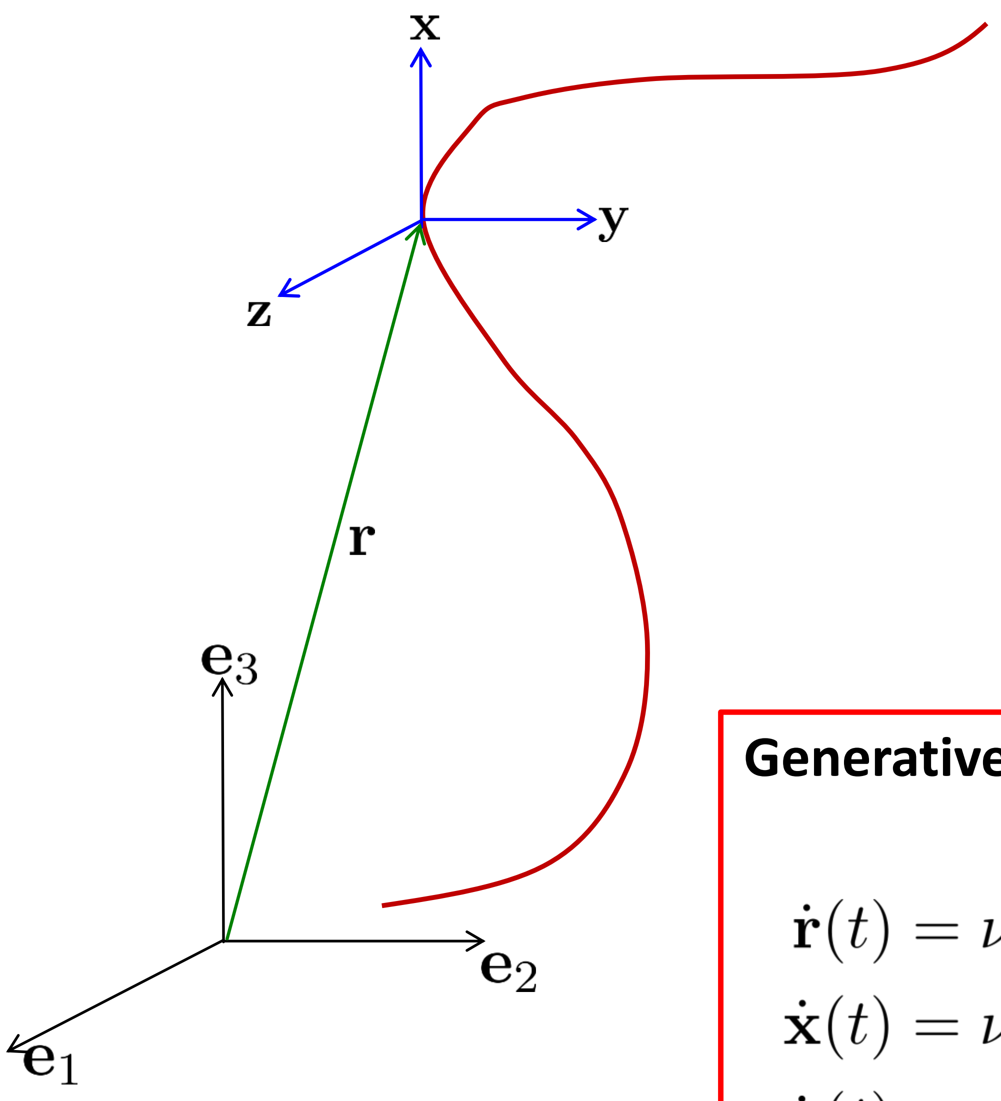
Outline

- Generative Models
- Trajectory Reconstruction
  - Regularized Inversion and Cross-Validation
  - Nonlinear Optimization – Mathematical Programming
  - Linear Quadratic Optimal Control – Jerk Minimization
  - Nonlinear Optimization – Pontryagin’s Maximum Principle (Ongoing Work)
- Analysis of Foraging in Echolocating Bats
- A glimpse of Flock Reconstruction

Self Steering Particle



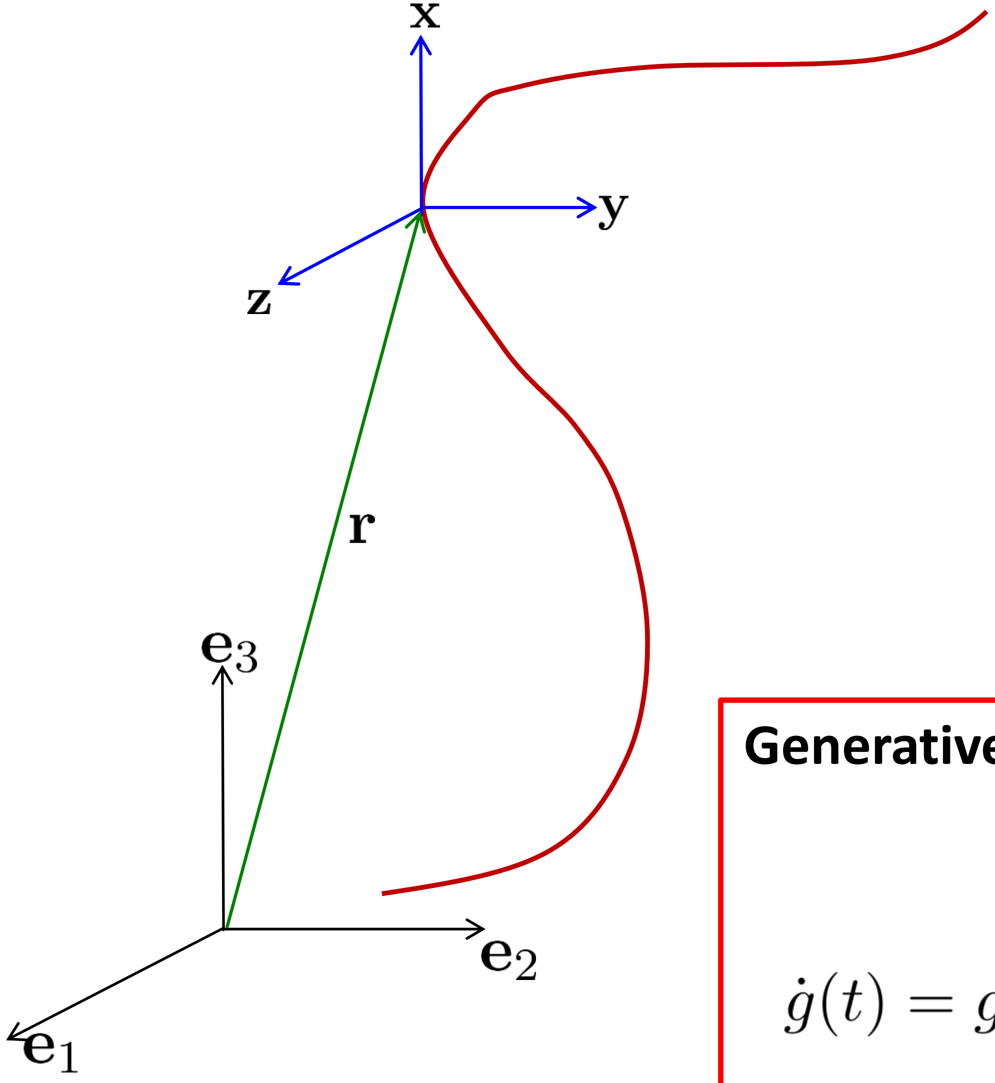
- Position vector: $\mathbf{r}$
- Natural Frenet frame: $[\mathbf{x} \ \ \mathbf{y} \ \ \mathbf{z}]$

- Unit tangent vector: $\mathbf{x} = \dfrac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$
- $[\mathbf{y} \ \ \mathbf{z}]$: Unit vectors spanning the plane perpendicular to the unit tangent vector.
- Speed of the trajectory: $\nu = |\dot{\mathbf{r}}|$
- Natural curvatures for the trajectory: u and v

Generative Model (Natural Frame Equations):

$$
\begin{aligned}
\dot{\mathbf{r}}(t) &= \nu(t)\mathbf{x}(t) \\
\dot{\mathbf{x}}(t) &= \nu(t)\left(u(t)\mathbf{y}(t) + v(t)\mathbf{z}(t)\right) \\
\dot{\mathbf{y}}(t) &= -\nu(t)u(t)\mathbf{x}(t) \\
\dot{\mathbf{z}}(t) &= -\nu(t)v(t)\mathbf{x}(t),
\end{aligned}
$$

Self Steering Particle



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Generative Model (Natural Frame Equations):

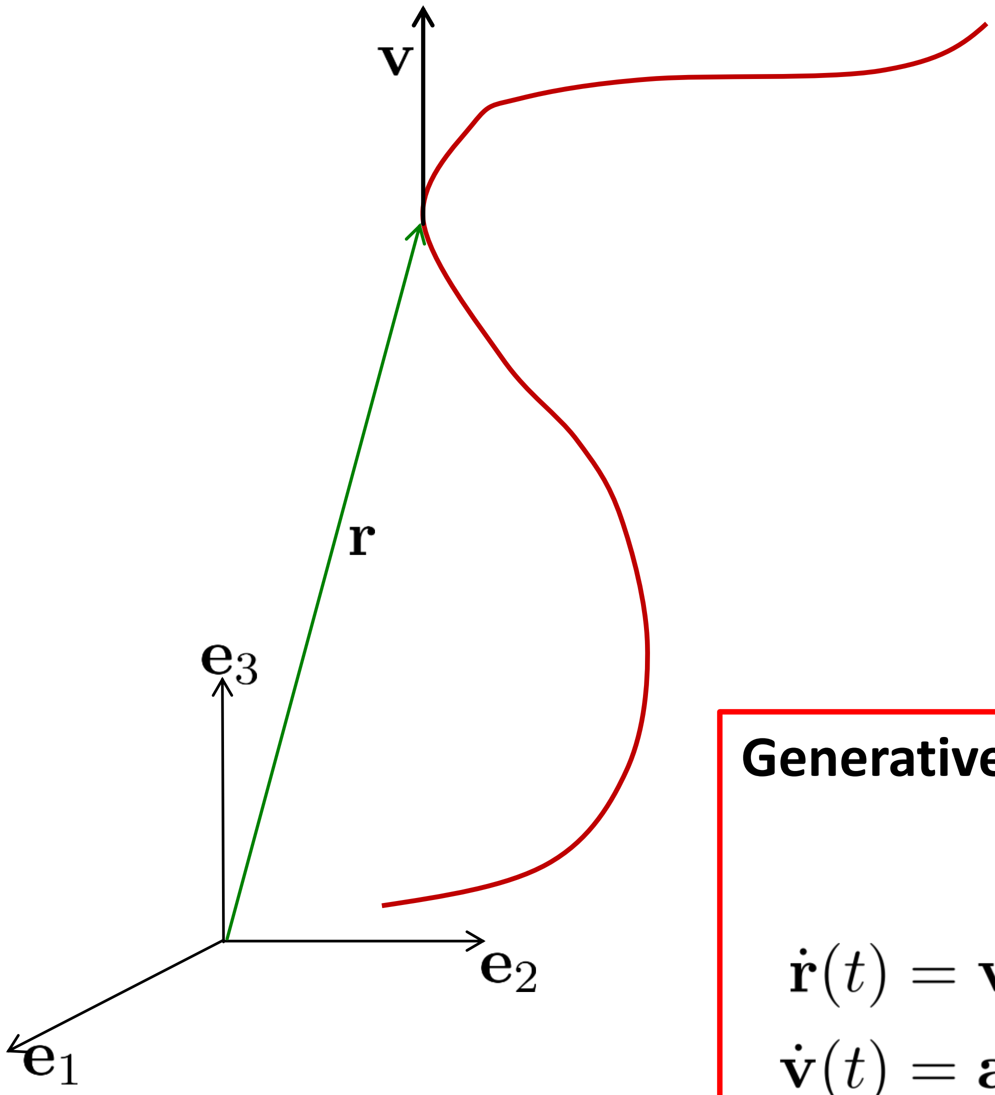
$$\dot{g}(t) = g(t)\xi(t)$$
$$\mathbf{r}(t) = [e_1 \;\; e_2 \;\; e_3]^T g e_4$$

with

$$g(t) = \begin{bmatrix} [\mathbf{x}(t) \;\; \mathbf{y}(t) \;\; \mathbf{z}(t)] & \mathbf{r}(t) \\ 0 & 1 \end{bmatrix} \in SE(3)$$

$$\xi = \nu \begin{bmatrix} 0 & -u & -v & 1 \\ u & 0 & 0 & 0 \\ v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in se(3)$$

Triple Integrator Model

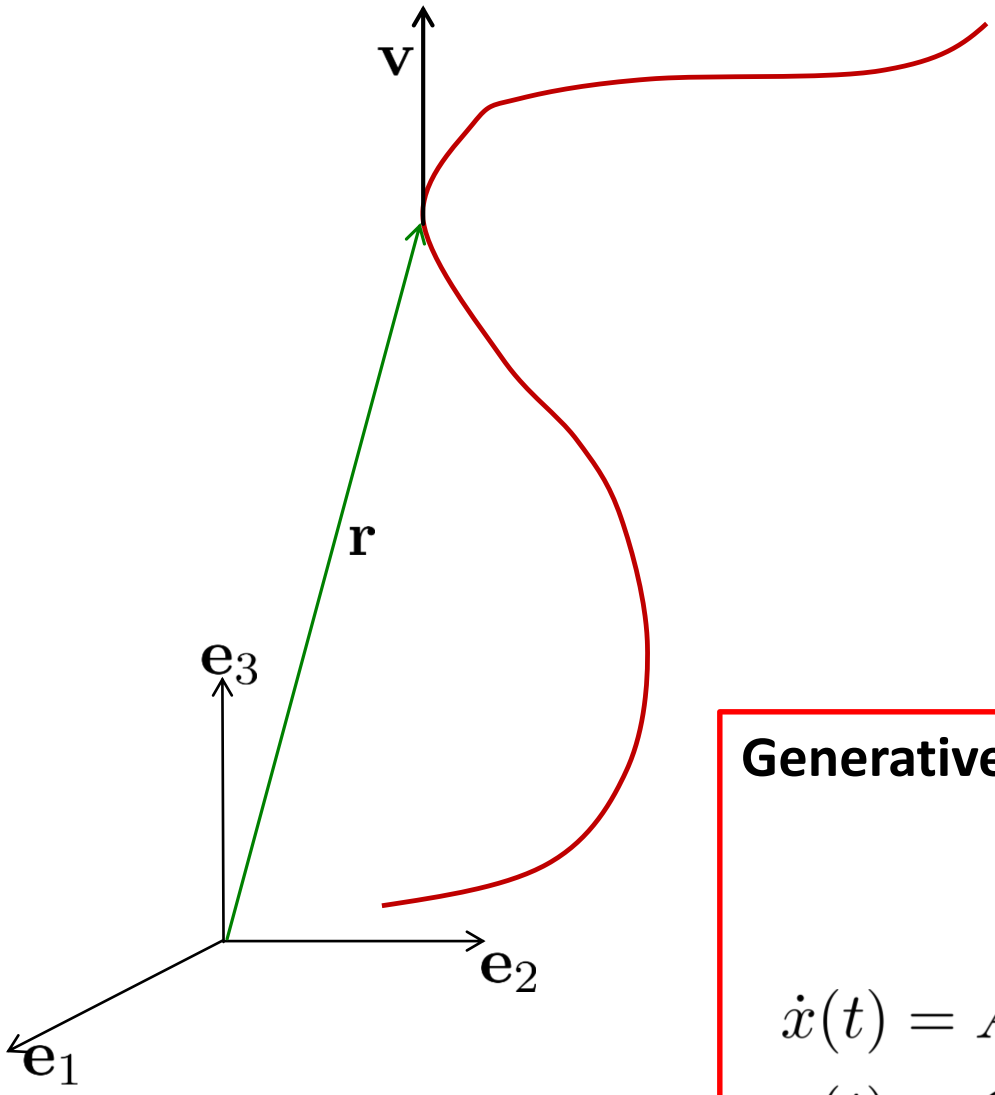


- Position vector: $\mathbf{r}$
- Velocity: $\mathbf{v}$
- Acceleration: $\mathbf{a}$
- Jerk (derivative of acceleration): $\mathbf{u}$

Generative Model (Triple Integrator):

$$\dot{\mathbf{r}}(t) = \mathbf{v}(t)$$
$$\dot{\mathbf{v}}(t) = \mathbf{a}(t)$$
$$\dot{\mathbf{a}}(t) = \mathbf{u}(t)$$

Triple Integrator Model



- Position vector: $\mathbf{r}$
- Velocity: $\mathbf{v}$
- Acceleration: $\mathbf{a}$
- Jerk (derivative of acceleration): $\mathbf{u}$

Generative Model (Triple Integrator):

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

with

$$x(t) = \begin{bmatrix} \mathbf{r}(t) \\ \mathbf{v}(t) \\ \mathbf{a}(t) \end{bmatrix}, \quad u(t) = \mathbf{u}(t), \quad y(t) = \mathbf{r}(t)$$

$$A = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \quad C = \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}^T$$

Relationship between the models

Self steering particle model *to* Triple integrator model

$$\mathbf{v} = \nu \mathbf{x}$$

$$\mathbf{a} = \dot{\nu}\mathbf{x} + u\nu^2\mathbf{y} + v\nu^2\mathbf{z}$$

$$\mathbf{u} = (\ddot{\nu} - \nu^3(u^2 + v^2))\mathbf{x} + (3u\nu\dot{\nu} + \dot{u}\nu^2)\mathbf{y} + (3v\nu\dot{\nu} + \dot{v}\nu^2)\mathbf{z}$$

Triple integrator model *to* Self steering particle model

$$\nu = \|\mathbf{v}\|$$

$$\mathbf{x} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

$$\dot{\mathbf{x}} = \frac{1}{\nu}(\mathbf{a} - (\mathbf{a}\cdot\mathbf{x})\mathbf{x})$$

$$\kappa = \frac{\|\dot{\mathbf{x}}\|}{\nu}$$

$$\tau = \frac{\mathbf{v}\cdot(\mathbf{a}\times\mathbf{u})}{\|\mathbf{v}\times\mathbf{a}\|^2}$$

- $u, v, \mathbf{y}, \mathbf{z}$ can be obtained by

$$u(t) = \kappa\cos\left(\theta_0 + \int_0^t \tau(\sigma)d\sigma\right)$$

$$v(t) = \kappa\sin\left(\theta_0 + \int_0^t \tau(\sigma)d\sigma\right)$$

$$\mathbf{y}(t) = \mathbf{y}(0) - \int_0^t \nu(\sigma)u(\sigma)\mathbf{x}(\sigma)d\sigma$$

$$\mathbf{z}(t) = \mathbf{z}(0) - \int_0^t \nu(\sigma)v(\sigma)\mathbf{x}(\sigma)d\sigma$$

Trajectory Reconstruction as an Optimization Problem

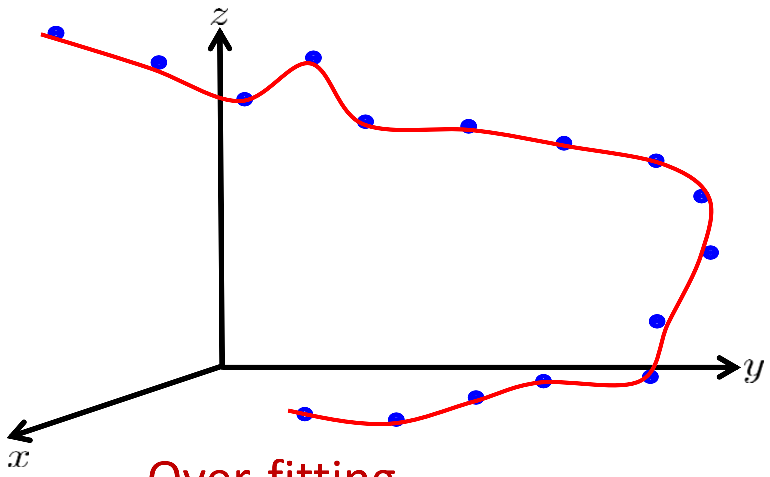
Regularized Inversion

Objective: Given a time series of noisy position data, reconstruct a trajectory to fit the data points.
Issues: The inverse problem is ill-posed.

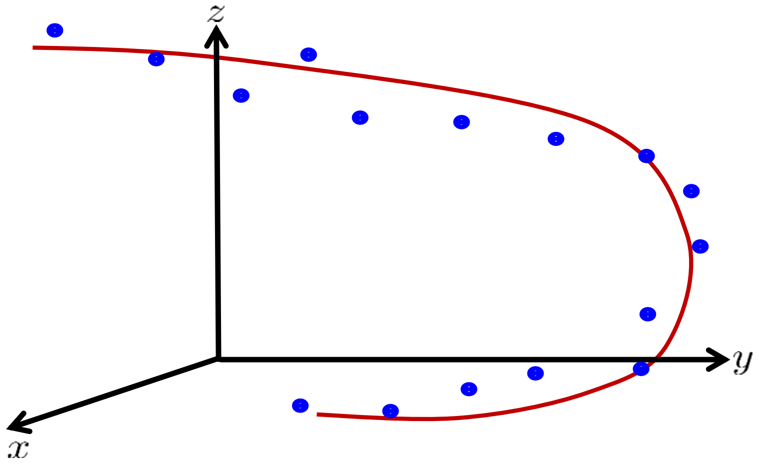
- Naive solution is highly sensitive to noise.
- Non-unique.

Solution: Introduce regularization (by adding a penalty term for lack of smoothness of the trajectory).

How can we determine an optimal balance between goodness of fit and smoothness of the reconstructed trajectory?



Over-fitting



Under-fitting

Ordinary Cross Validation !!!

Ordinary Cross Validation (OCV)

- Separate data set into two disjoint subsets:
  - Estimation subset
  - Validation subset
- *Estimation subset* is used for trajectory reconstruction.
- *Validation subset* is used to evaluate the performance of the reconstruction (*fit-error*).
- All possible estimation subsets are considered to avoid any local bias.
- An optimal amount of regularization maximizes the performance over validation subset.
- We adopt leaving-one-out strategy in our line of works.

Regularized Inversion – Nonlinear Optimization (Mathematical Programming view)

Observed Positions

$\{r_i\}_{i=0}^{N}$ → **Regularized Inversion** → $\mathbf{r}, [\mathbf{x}\ \mathbf{y}\ \mathbf{z}];\quad u, v, \nu$

State; Controls

Dynamics

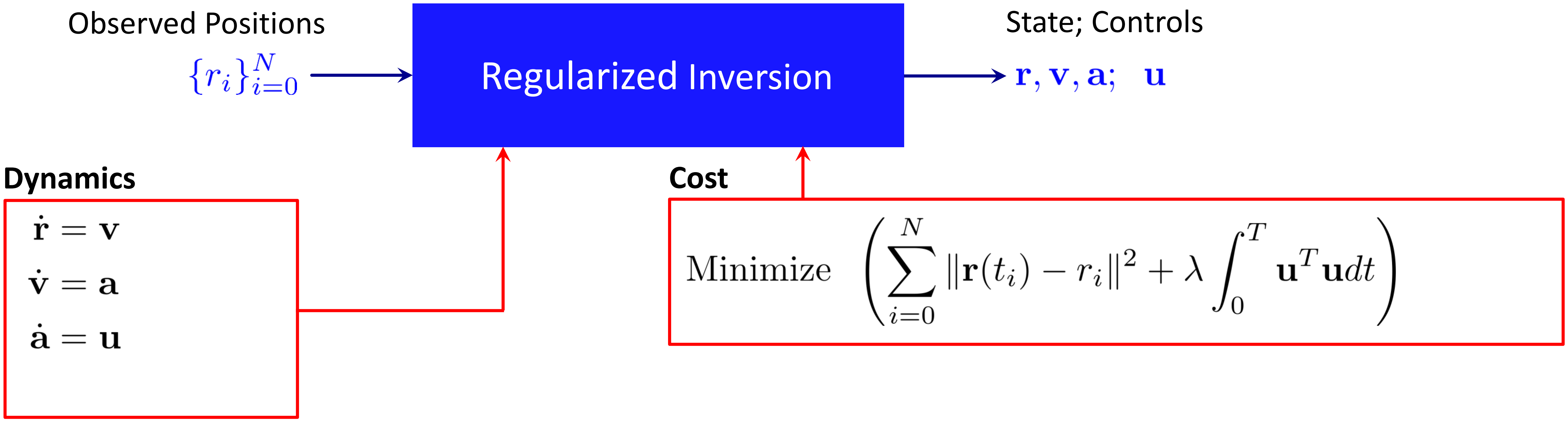
$$\dot{\mathbf{r}} = \nu \mathbf{x}$$
$$\dot{\mathbf{x}} = \nu\left(u\mathbf{y} + v\mathbf{z}\right)$$
$$\dot{\mathbf{y}} = -\nu u \mathbf{x}$$
$$\dot{\mathbf{z}} = -\nu v \mathbf{x}$$

Cost

$$\text{Minimize}\quad \left(\sum_{i=0}^{N} \|\mathbf{r}(t_i) - r_i\|^2 + \lambda \int_0^T (\dot{u}^2 + \dot{v}^2 + \dot{\nu}^2)dt\right)$$

- The problem is solved numerically over a restricted search space of piecewise constant functions (Matlab: *fminunc*).
- Reparametrizations (Cayley transform, exponential function) have been used to transform this problem into an optimization problem over a high-dimensional Cartesian space.
- However, this algorithm is capable of estimating curvature with higher resolution (sub-frame adaptable).
- This approach is computationally very demanding.
- It may get stuck at a local optimum.

Regularized Inversion – Linear Approach



- Integrability theory of linear-quadratic optimal control can be used to obtain an analytic solution.
- Regularization, in this case, penalizes high values of the jerk path integral.
- The 2/3-power law can be interpreted as a consequence of the minimization of jerk path integral.

Regularized Inversion – Linear Approach – Linear Quadratic Optimal Control

$$\underset{x(t_0),u}{\text{Minimize}} \quad J(x(t_0),u) = \sum_{i=0}^{N} \|y(t_i) - r_i\|^2 + \lambda \int_0^T u^T u dt$$

subject to **System dynamics** : $\dot{x} = Ax + Bu, \quad y = Cx$

$x(t_0) \in \mathbb{R}^n, \quad u \in \mathcal{U}$

- Apply path independence lemma.
  - Symmetric bilinear form $K : [0,T] \to \mathbb{R}^{n\times n}$
  - Linear functional $\eta : [0,T] \to \mathbb{R}^n$

$$\dot{K}(t) = -A^T K(t) - K(t)A + K(t)BB^T K(t),$$
$$K(t_N^+) = 0,$$
$$K(t_i^+) - K(t_i^-) = -\frac{1}{\lambda} C^T C.$$

$$\dot{\eta}(t) = -\left(A^T - K(t)BB^T\right)\eta(t),$$
$$\eta(t_N^+) = 0,$$
$$\eta(t_i^+) - \eta(t_i^-) = \frac{2}{\lambda} C^T r_i.$$

- **Optimal Control Input:**

$$u_{opt}(t) = -B^T \left(K(t)x(t) + \frac{1}{2}\eta(t) \right)$$

- **Optimal Initial Condition:**

$$\left[K(t_0^-)\right] x_{opt}(t_0) + \frac{1}{2}\eta(t_0^-) = 0$$ ← Solvability??

Regularized Inversion – Linear Approach – Existence of optimal initial condition

Optimal Initial Condition:

$$\left[K(t_0^-)\right] x_{opt}(t_0) + \frac{1}{2}\eta(t_0^-) = 0$$

Proposition 3.1:*

The solution of the Riccati equation assumes the form

$$K(t_i^-) = \frac{1}{\lambda}\sum_{k=i}^{N} \Phi_{\Sigma}(t_i, t_k) C^T C \Phi_{\Sigma}^T(t_i, t_k)$$

for any $i \in \{0, 1, \cdots, N\}$ where $\Sigma(t) = -(A - \frac{1}{2}BB^T K(t))^T$, and Φ_{Σ} is the transition matrix for Σ.

Theorem 3.5:*

For the trajectory reconstruction problem, the optimal initial condition is uniquely solvable for almost any time index set $\{t_i\}_{i=0}^{N}$.

Proposition 3.2:*

$$\eta(t_i^+) = -\frac{2}{\lambda}\sum_{k=i+1}^{N} \Phi_{\tilde{\Sigma}}(t_i, t_k) C^T r_k$$

$$\eta(t_i^-) = -\frac{2}{\lambda}\sum_{k=i}^{N} \Phi_{\tilde{\Sigma}}(t_i, t_k) C^T r_k$$

where $\tilde{\Sigma}(t) = -(A - BB^T K(t))^T$.

Proposition 3.4:*

$(-\Sigma^T, C)$ forms an observable pair for the trajectory reconstruction problem.

* B. Dey, P. S. Krishnaprasad, *Trajectory Smoothing as a Linear Optimal Control Problem*, Proc. 50th Annual Allerton Conference, 1490 - 1497, Monticello, IL, October 2012.

Regularized Inversion – Linear Approach – Co-state Based Approach

- Co-state Variable:

$$p(t) \triangleq K(t)x(t) + \frac{1}{2}\eta(t)$$

- Co-state Dynamics:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ p(t) \end{bmatrix} = \begin{bmatrix} A & -BB^T \\ 0 & -A^T \end{bmatrix}\begin{bmatrix} x(t) \\ p(t) \end{bmatrix}$$

- Boundary and Jump Conditions:

$$p(t_i^+) - p(t_i^-) = \frac{1}{\lambda}C^T\left(r_i - Cx(t_i)\right)$$

$$p(t_N^+) = p(t_0^-) = 0$$

$$\begin{bmatrix} x(t_{i+1}) \\ p(t_{i+1}^+) \end{bmatrix} = \underbrace{\begin{bmatrix} e^{A\Delta_i} & -e^{A\Delta_i}\mathbb{W}_i \\ -\frac{1}{\lambda}C^TCe^{A\Delta_i} & \left[e^{-A^T\Delta_i} + \frac{1}{\lambda}C^TCe^{A\Delta_i}\mathbb{W}_i\right] \end{bmatrix}}_{\Lambda_i}\begin{bmatrix} x(t_i) \\ p(t_i^+) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{\lambda}C^T \end{bmatrix}}_{\Gamma} r_{i+1}$$

where, $\mathbb{W}_i = \int_0^{t_{i+1}-t_i} e^{-A\tau}BB^Te^{-A^T\tau}d\tau$

$$[0 \;\; I]\left(\prod_{i=0}^{N-1}\Lambda_i\right)\begin{bmatrix} I \\ -\frac{1}{\lambda}C^TC \end{bmatrix}x(t_0) = -[0 \;\; I]\sum_{i=0}^{N}\left(\prod_{j=i}^{N-1}\Lambda_j\right)\Gamma r_i$$

Regularized Inversion – Linear Approach – OCV

- Let $\{x_{opt}^{[\lambda,k]}, u^{[\lambda,k]}\}$ be a minimizer of:

$$\sum_{\substack{i=0 \\ i\neq k}}^{N} \|\mathbf{r}(t_i) - r_i\|^2 + \lambda \int_0^T u^T u dt$$

- The corresponding reconstructed trajectory is $\mathbf{r}^{[\lambda,k]}(\cdot)$.

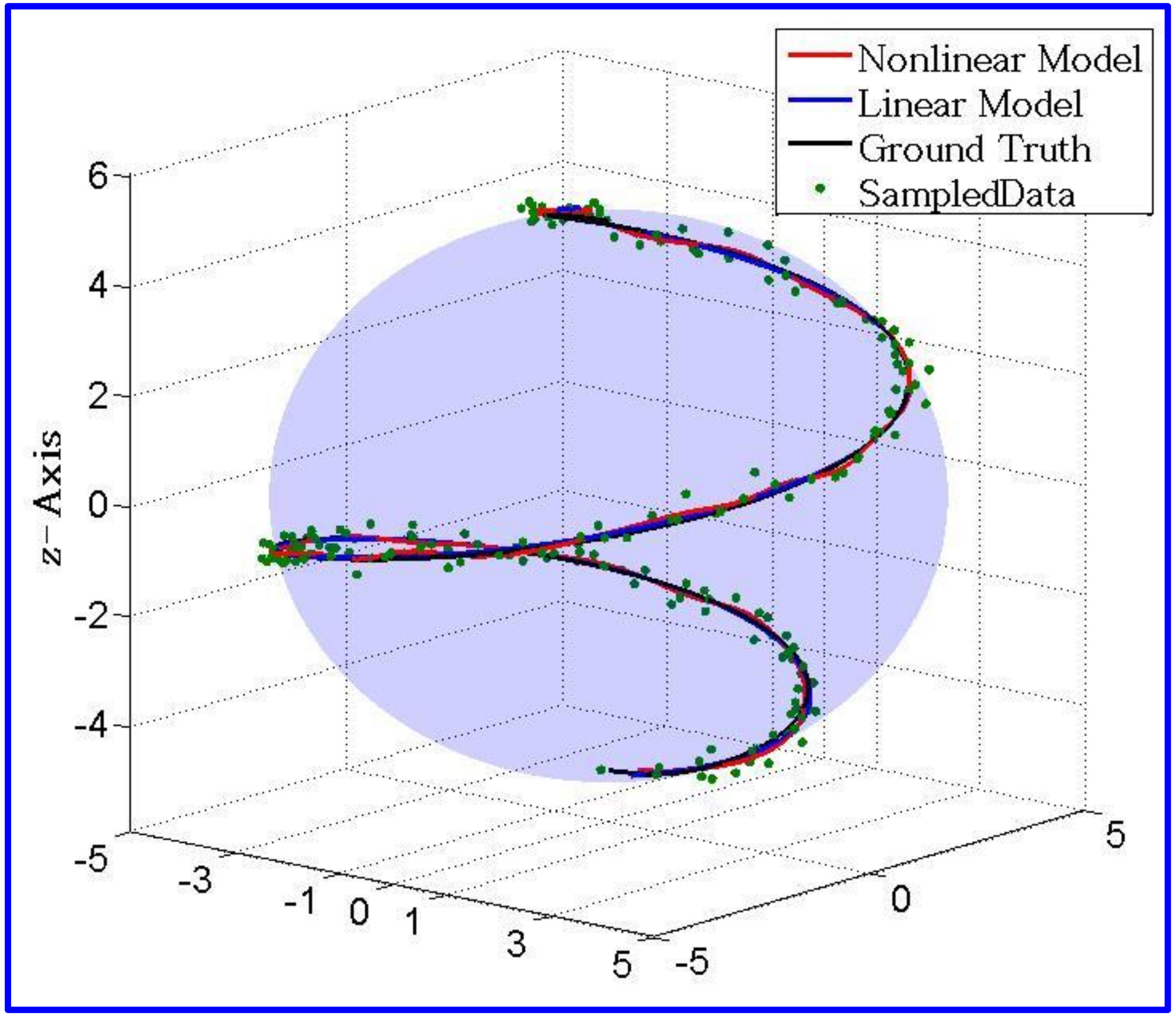
- Then, the OCV Cost is defined as:

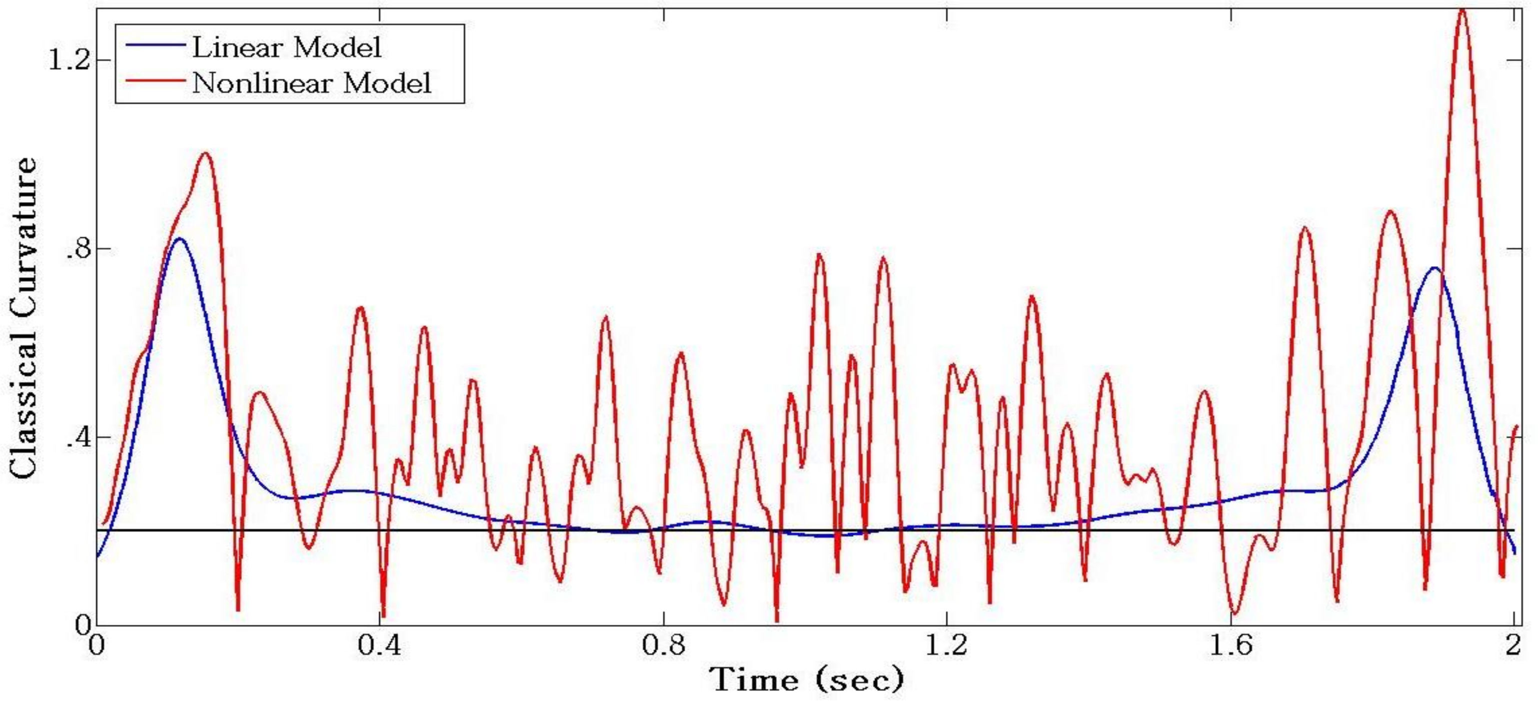
$$V_0(\lambda) = \frac{1}{N+1} \sum_{k=0}^{N} \|\mathbf{r}^{[\lambda,k]}(t_k) - r_k\|^2$$

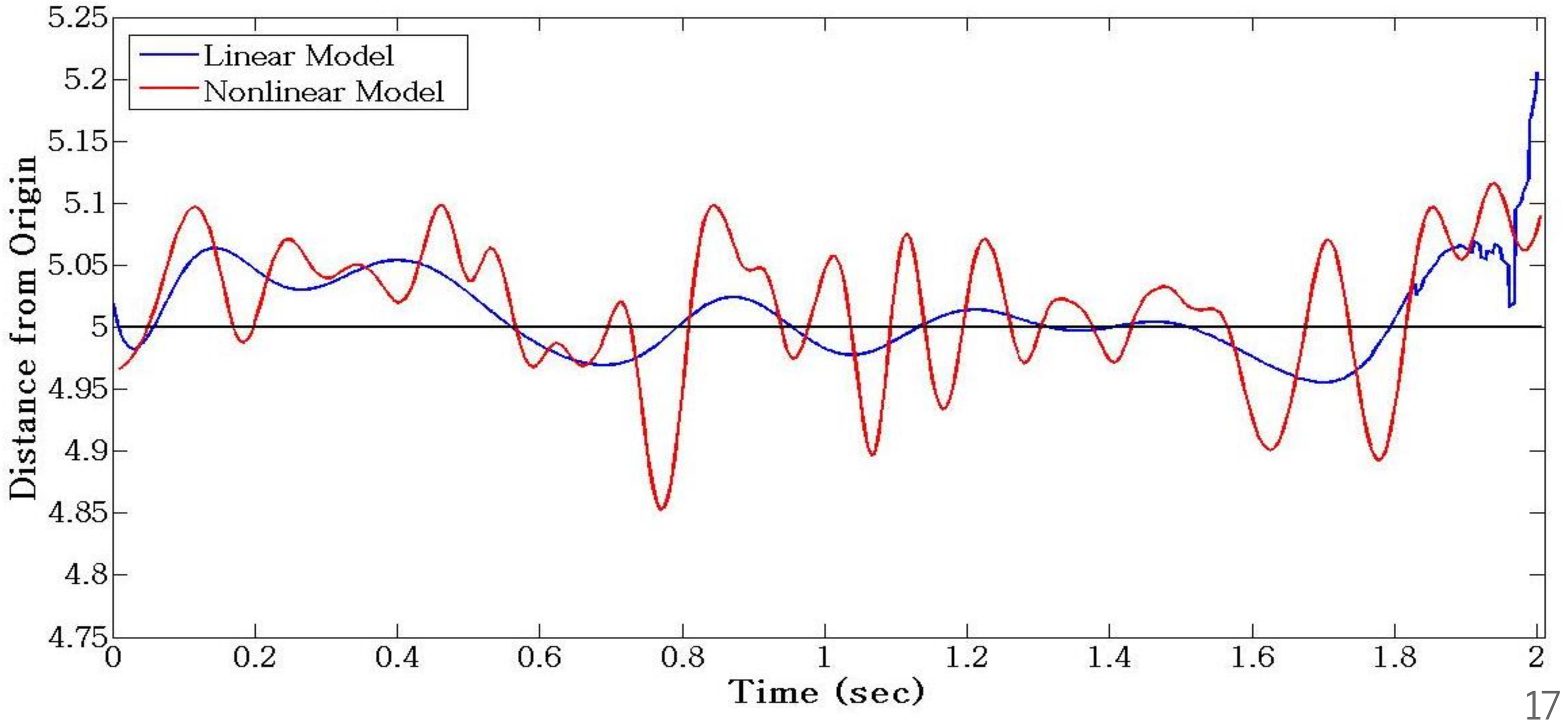
- Hence, OCV estimate for λ is defined as:

$$\lambda^* = \underset{\lambda \in \mathbb{R}_+}{\operatorname{argmin}} \; V_0(\lambda)$$

Regularized Inversion – Linear Approach – Numerical Results







Regularized Inversion – Nonlinear Optimization (Pontryagin's Maximum Principle)*

- **Optimal Control Problem:**

$$\underset{q(t_0),u}{\text{Minimize}} \quad \int_{t_0}^{t_N} L(q,u)dt + \sum_{i=0}^{N} F(q(t_i), q_i)$$

$$\text{subject to:} \quad \dot{q}(t) = f(q(t), u(t)), \quad q(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$$

- **Control/Pre Hamiltonian:**

$$H(q,p,u) = \langle p, f(q,u) \rangle - L(q,u)$$

- **Optimal Control Input:**

$$H(q^*(t), p(t), u^*(t)) = \underset{u}{\text{Max}} \; H(q^*(t), p(t), u)$$

- **Dynamics:**

$$\dot{q}^*(t) = \frac{\partial H}{\partial p}(q^*(t), p(t), u^*(t))$$

$$\dot{p}(t) = -\frac{\partial H}{\partial q}(q^*(t), p(t), u^*(t))$$

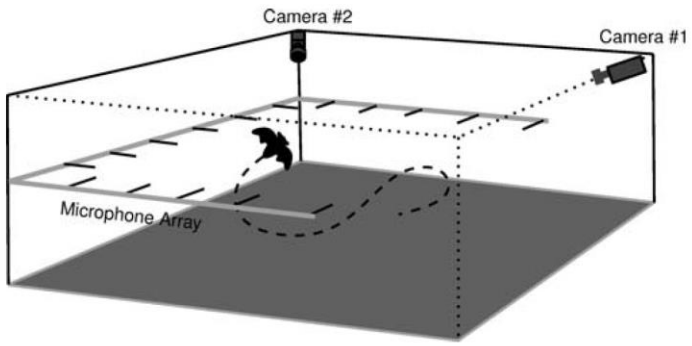
- **Boundary and Jump Conditions:**

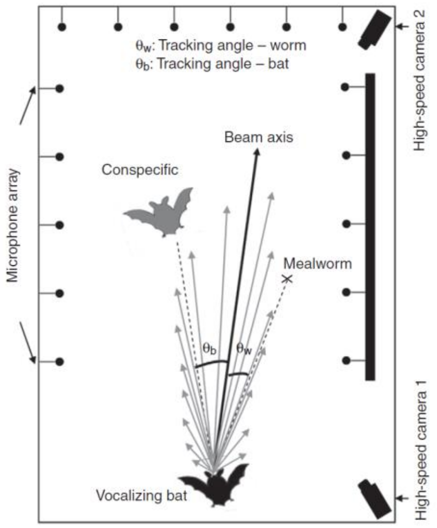
$$p(t_0^-) = p(t_N^+) = 0, \qquad p(t_i^+) - p(t_i^-) = \left. \frac{\partial F}{\partial q(t_i)} \right|_{q^*}$$

* B. Dey, P. S. Krishnaprasad, *Control-Theoretic Data Smoothing*, CDC 2014 (*8:30 AM, December 17, Wednesday*, Session: Optimal Control II, Room: Georgia 2).

Analysis of Foraging in Echolocating Bats

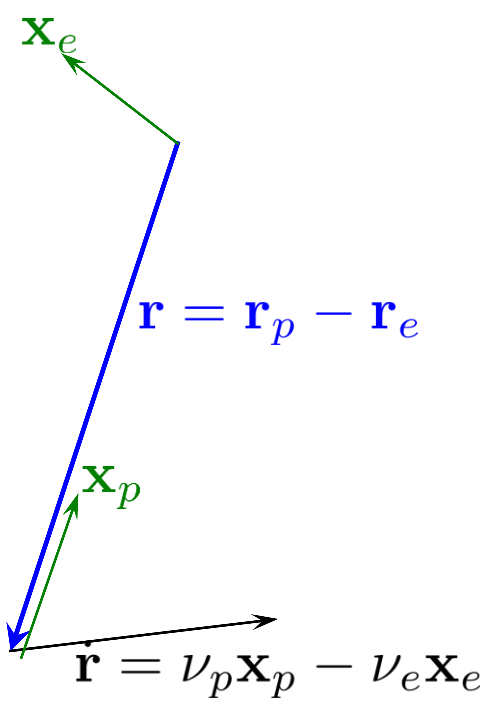
(Collaboration with Cynthia Moss)





Bat Flight – Strategies and Segmentation

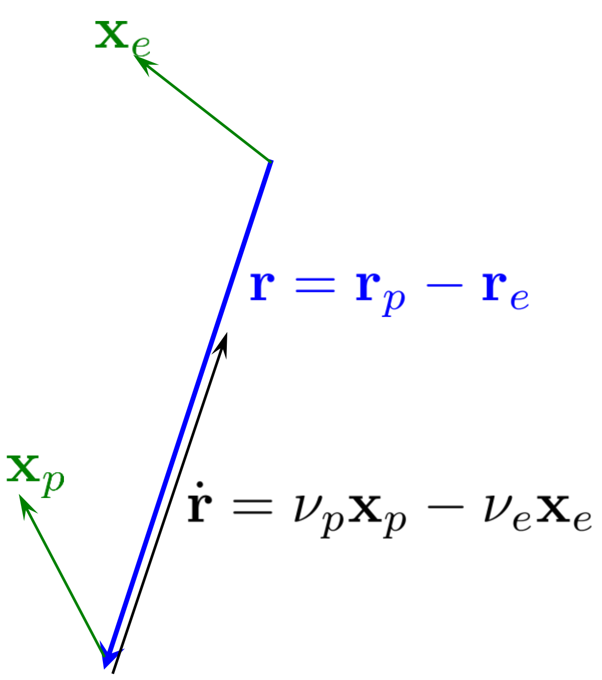
- **Classical Pursuit (CP)**



Contrast function

$$\Lambda = \mathbf{x}_p \cdot \frac{\mathbf{r}}{|\mathbf{r}|}$$

- **Motion Camouflage (MC/CATD)**



Contrast function

$$\Gamma = \frac{\mathbf{r}}{|\mathbf{r}|} \cdot \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$$

- Following Property

  Velocity vector ($\nu_p x_p$) has a negative projection on the baseline vector $\mathbf{r}$.

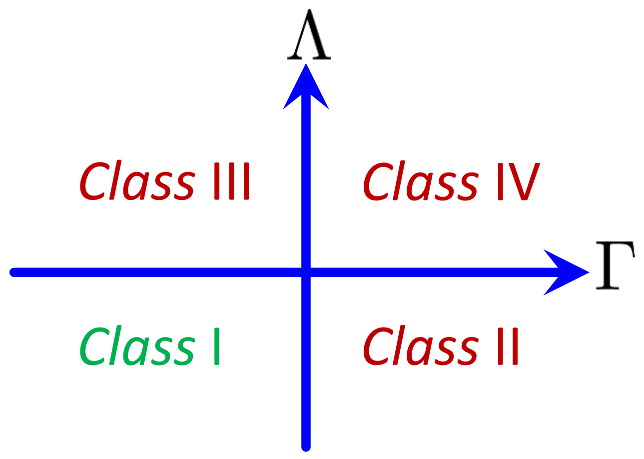
$$\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{x}_p < 0 \quad \Leftrightarrow \quad \Lambda < 0$$

- Convergence

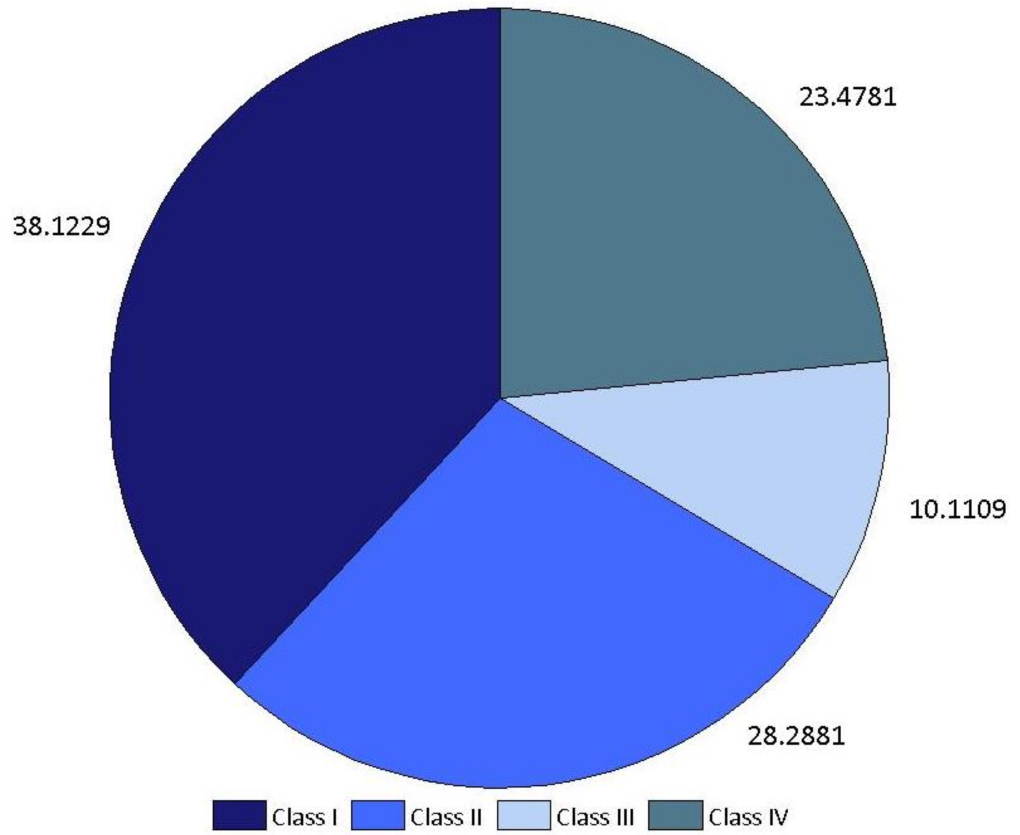
  Baseline vector ($\mathbf{r}$) has a shrinking length

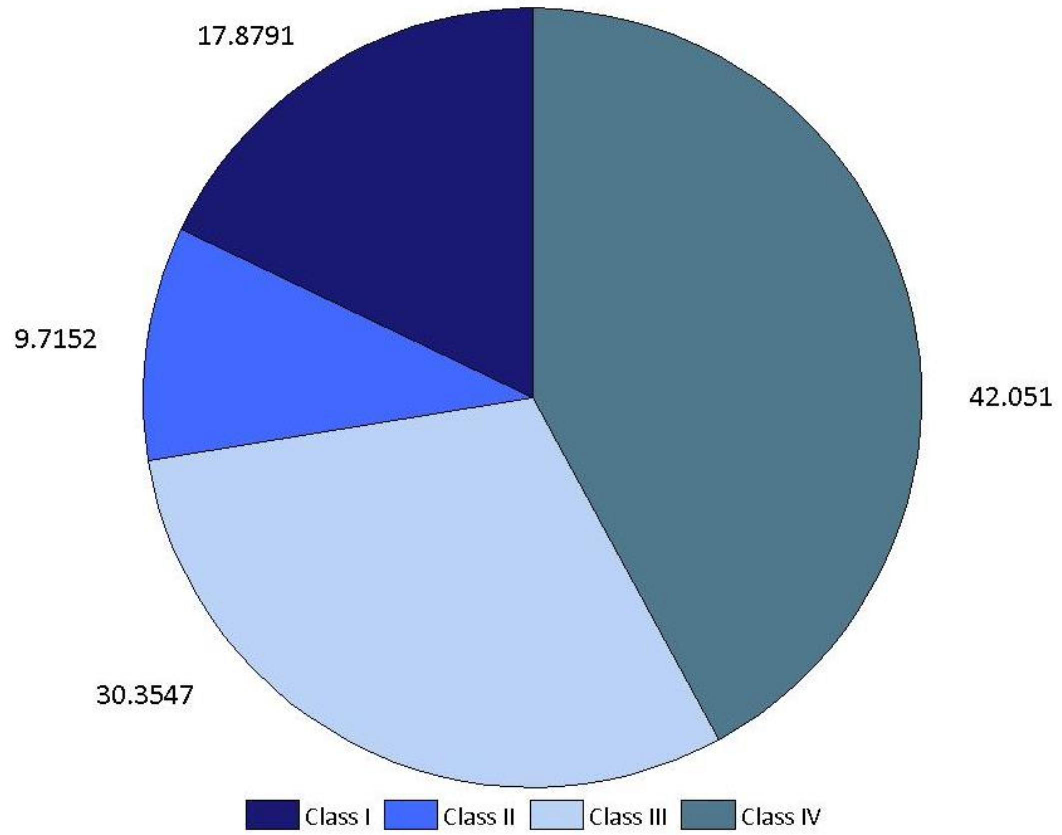
$$\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}} < 0 \quad \Leftrightarrow \quad \Gamma < 0$$

Bat Flight – Role Identification in Pursuit Events

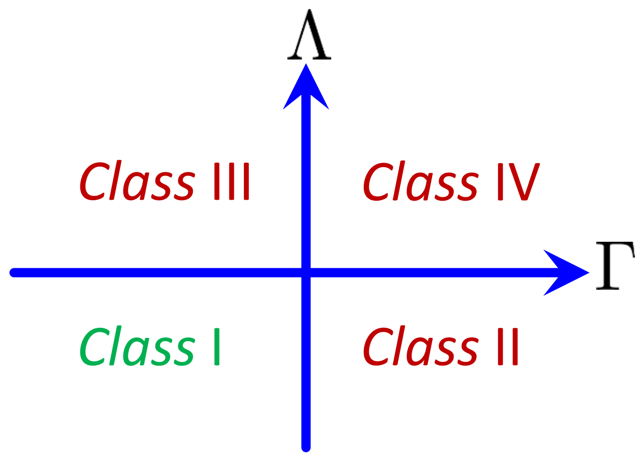


- Bat-Bat Interaction

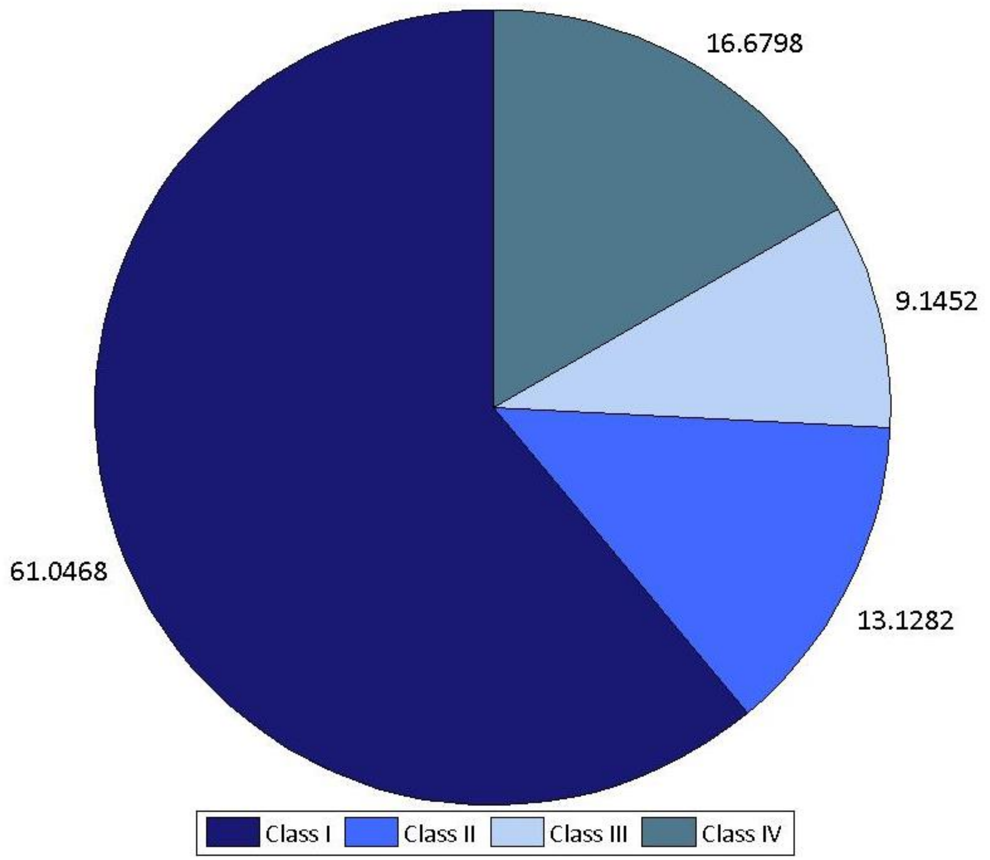




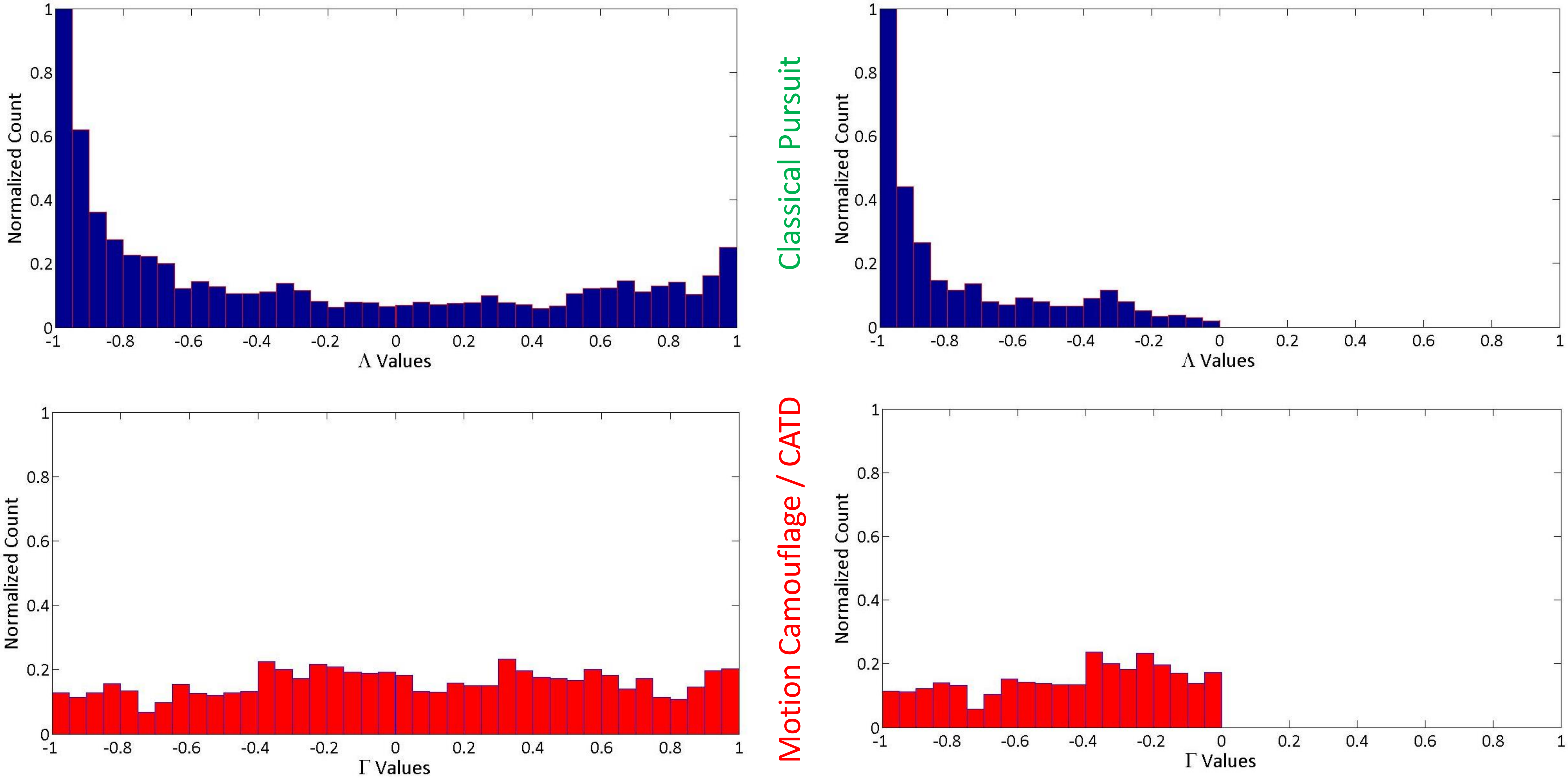
Bat Flight – Role Identification in Pursuit Events



- Bat-Insect (Praying Mantis) Interaction

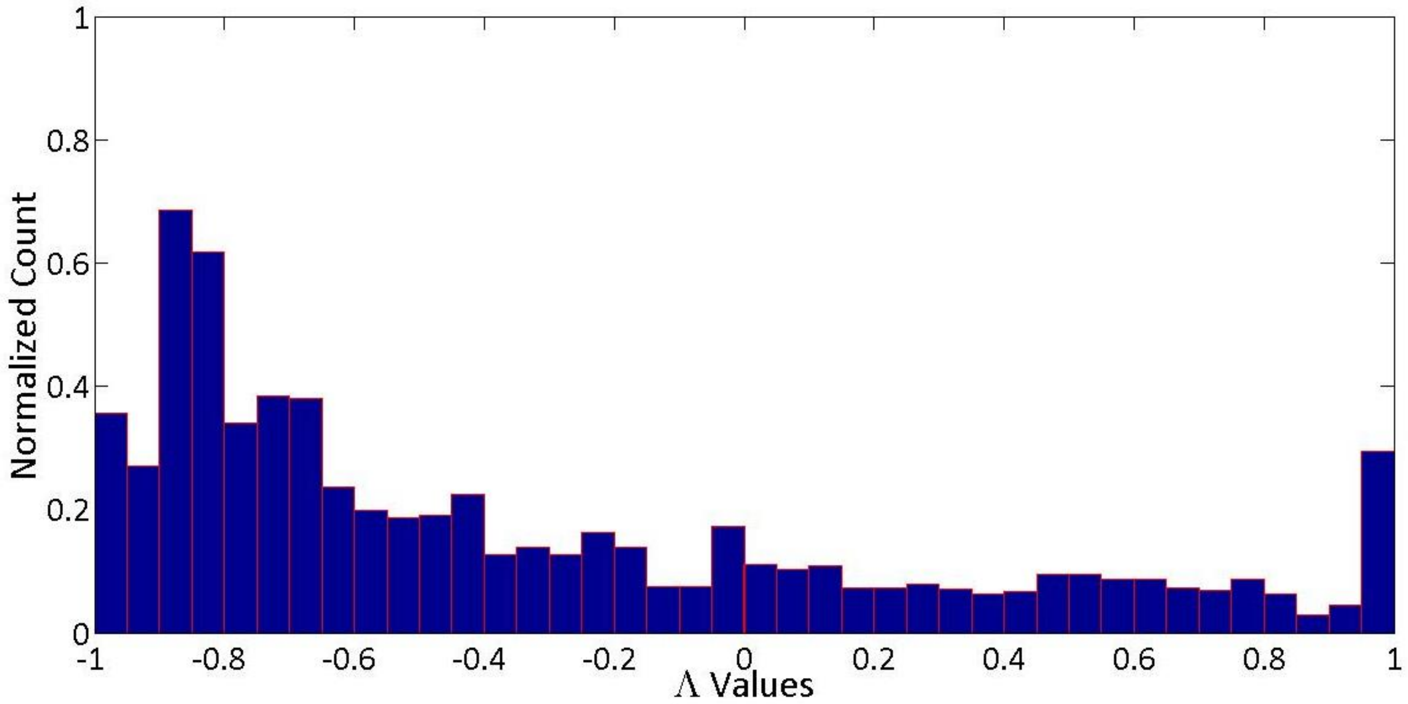


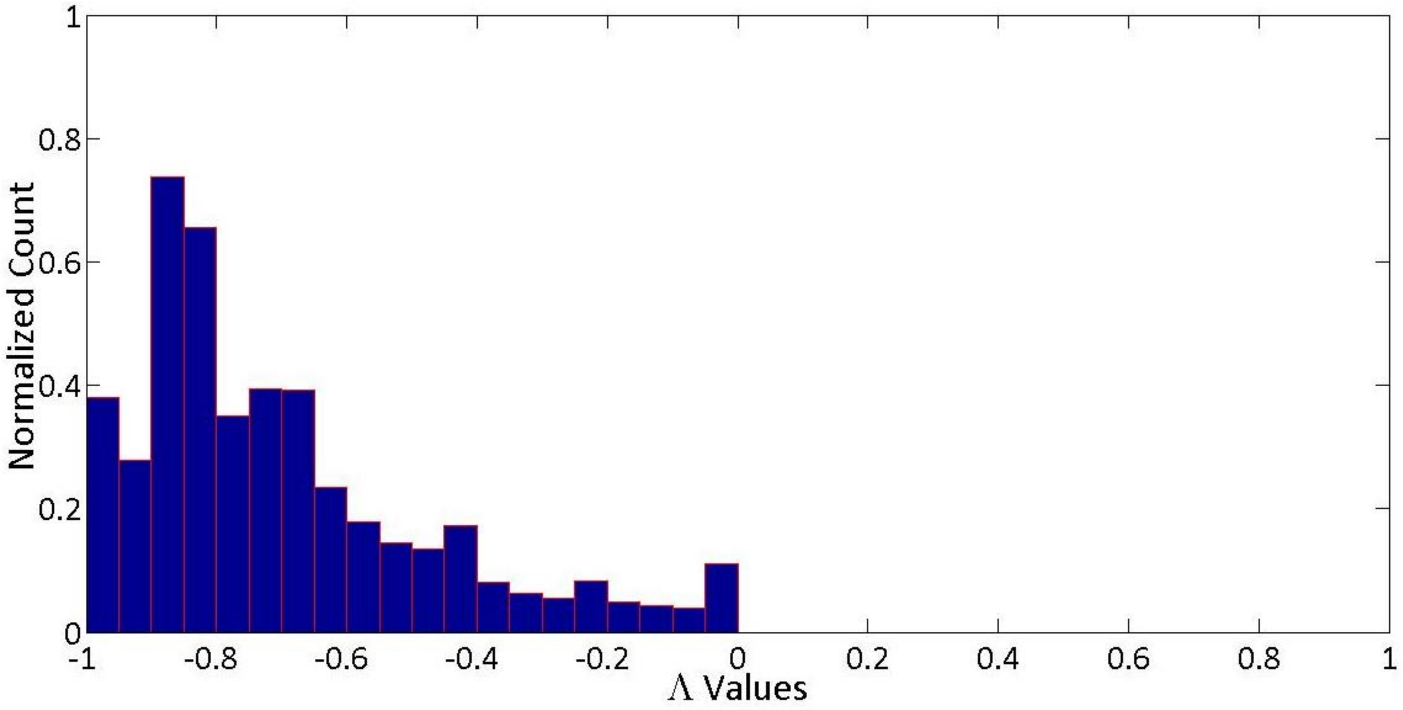
Bat Flight – Analysis of Contrast Functions – Bat-Bat Interaction



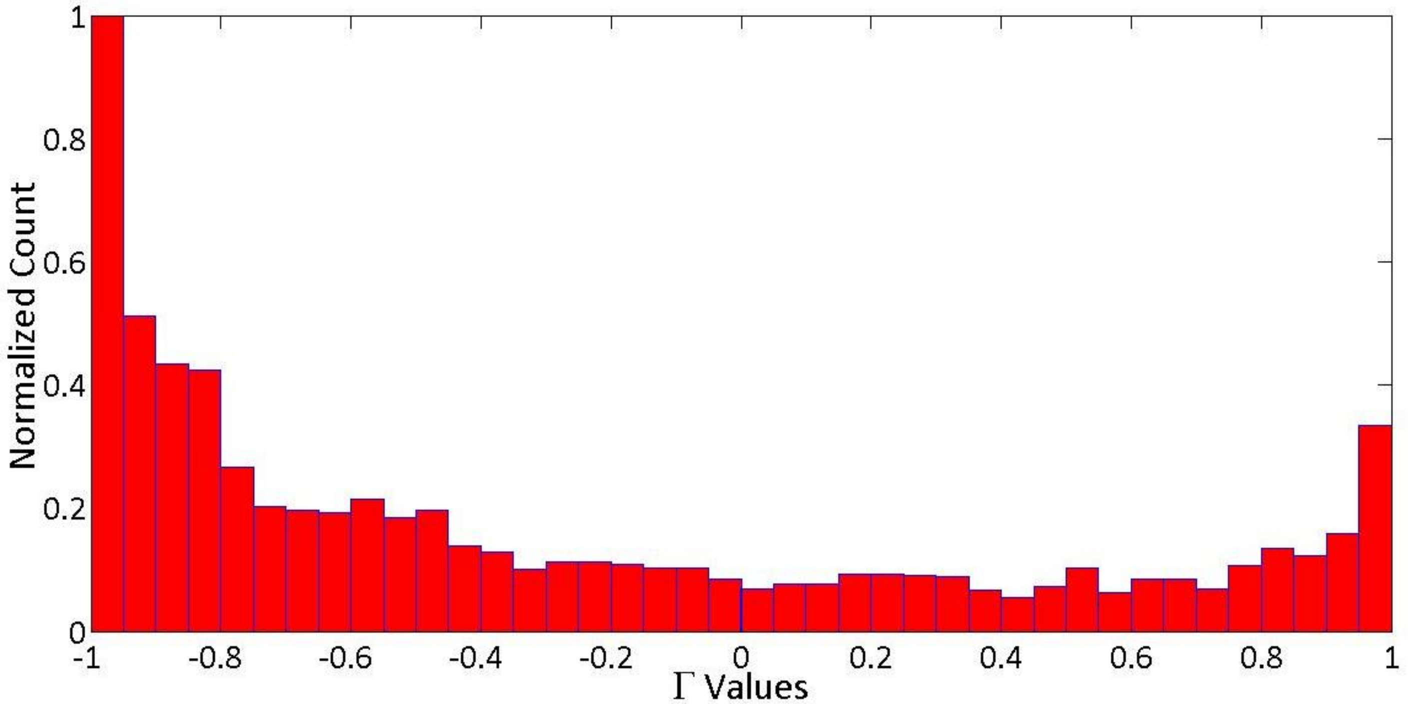
Bat Flight – Analysis of Contrast Functions – Bat-Insect Interaction

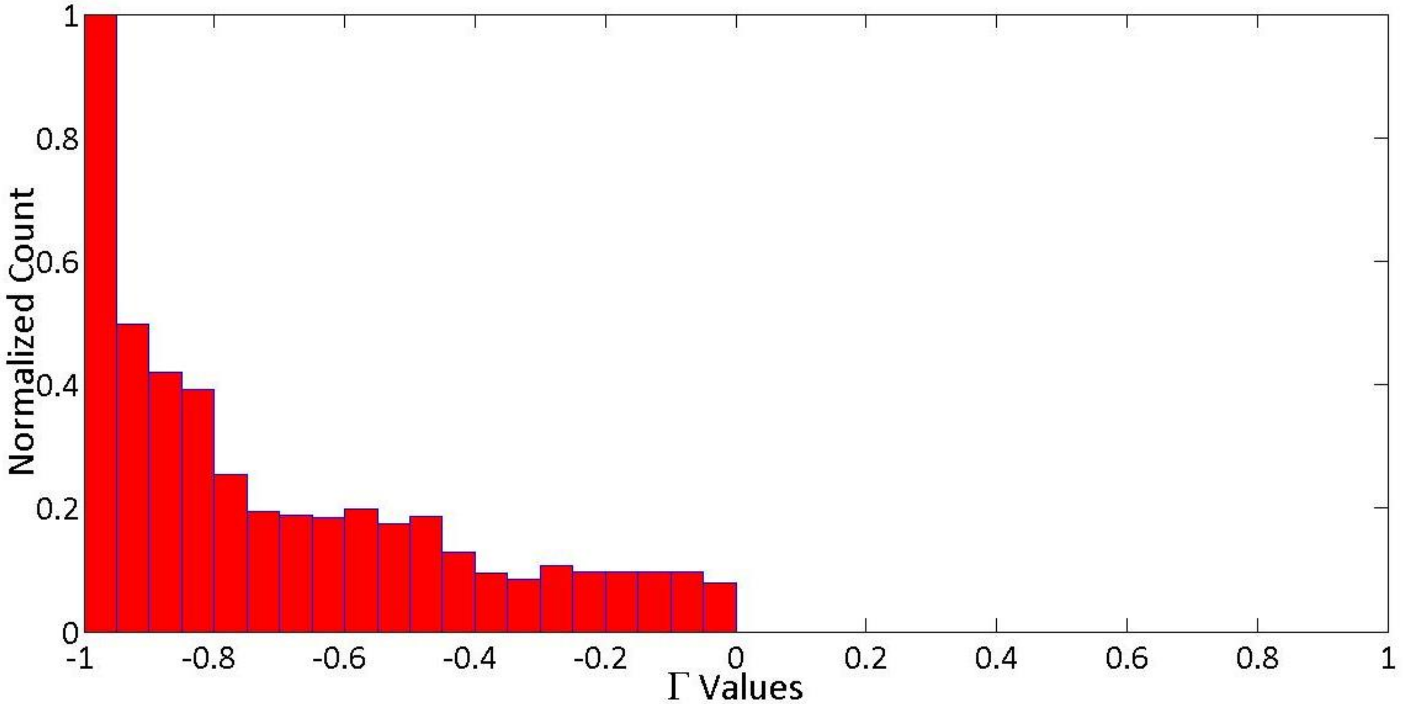
Classical Pursuit





Motion Camouflage / CATD





Bat Flight – Analysis of Steering Control

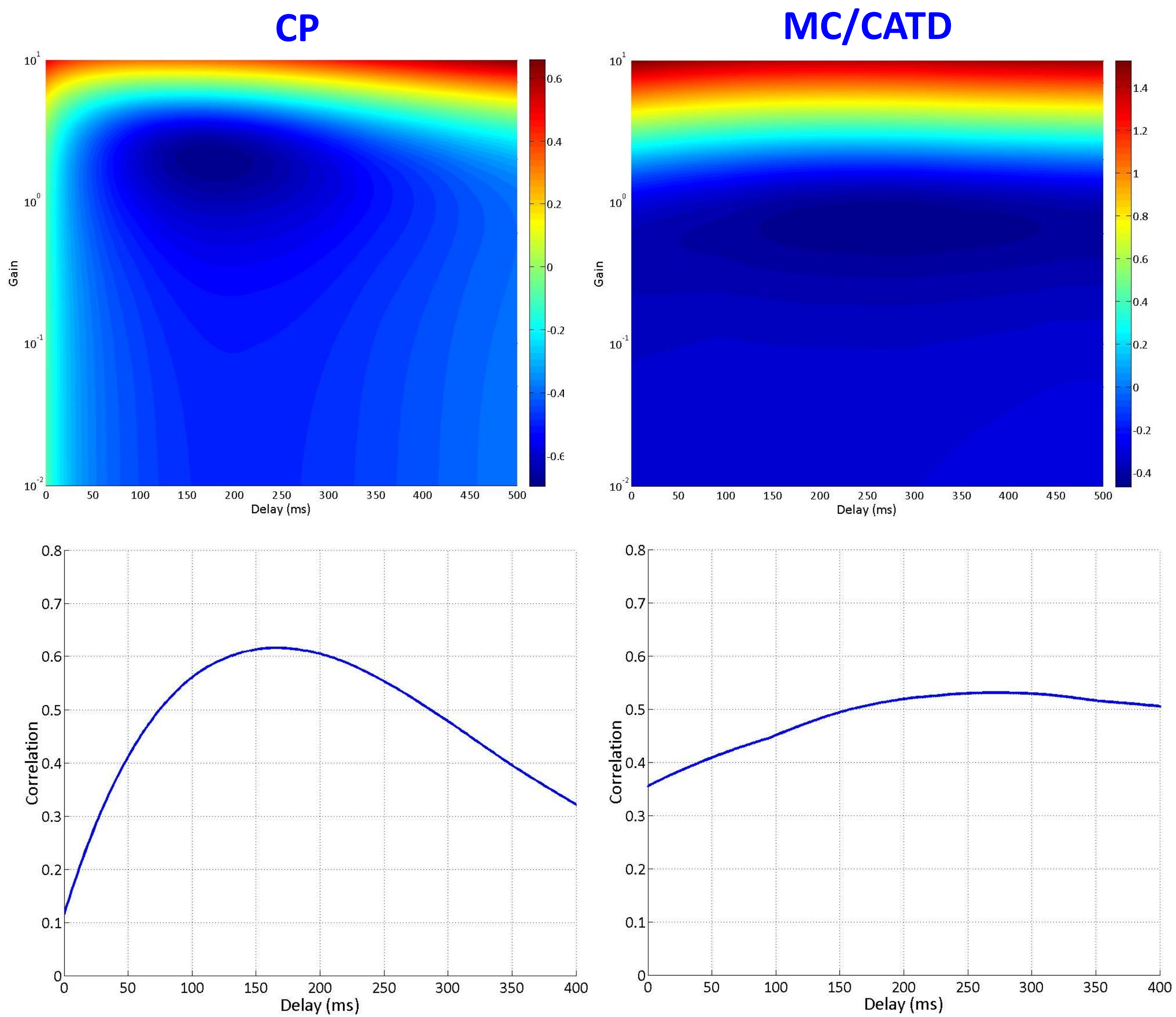
$$\underset{\mu>0,\delta\in\mathbb{N}}{\text{Minimize}}\left(\frac{1}{|\mathcal{E}|-\delta}\sum_{t_k\in\mathcal{E}}\left(\|u_{em}(t_k)-u_{th}(t_k-\delta)\|^2+\|v_{em}(t_k)-v_{th}(t_k-\delta)\|^2\right)\right)$$

$\Updownarrow \approx$

Strategy	Steering Feedback Law
Motion Camouflage [1,2] (MC/CATD)	$u_{th}=-\frac{\mu}{\nu_p}\left(\mathbf{z}_p\cdot\left(\dot{\mathbf{r}}\times\frac{\mathbf{r}}{\lvert\mathbf{r}\rvert}\right)\right)$ $v_{th}=\frac{\mu}{\nu_p}\left(\mathbf{y}_p\cdot\left(\dot{\mathbf{r}}\times\frac{\mathbf{r}}{\lvert\mathbf{r}\rvert}\right)\right)$
Classical Pursuit [3] (CP)	$u_{th}=-\frac{\mu}{\nu_p}\left(\mathbf{y}_p\cdot\frac{\mathbf{r}}{\lvert\mathbf{r}\rvert}\right)-\frac{1}{\nu_p\lvert\mathbf{r}\rvert}\left(\mathbf{z}_p\cdot\left(\dot{\mathbf{r}}\times\frac{\mathbf{r}}{\lvert\mathbf{r}\rvert}\right)\right)$ $v_{th}=-\frac{\mu}{\nu_p}\left(\mathbf{z}_p\cdot\frac{\mathbf{r}}{\lvert\mathbf{r}\rvert}\right)+\frac{1}{\nu_p\lvert\mathbf{r}\rvert}\left(\mathbf{y}_p\cdot\left(\dot{\mathbf{r}}\times\frac{\mathbf{r}}{\lvert\mathbf{r}\rvert}\right)\right)$

$$\underset{\delta\in\mathbb{N}}{\text{Maximize}}\ \text{Corr}\left(\begin{bmatrix}u_{em}\\ v_{em}\end{bmatrix},\begin{bmatrix}u_{th}^{\delta}\\ v_{th}^{\delta}\end{bmatrix}\right),\qquad u_{th}^{\delta}(k)=u_{th}(k-\delta),\ \ v_{th}^{\delta}(k)=v_{th}(k-\delta)$$

Bat Flight – Analysis of Steering Control – Bat-Bat Interaction

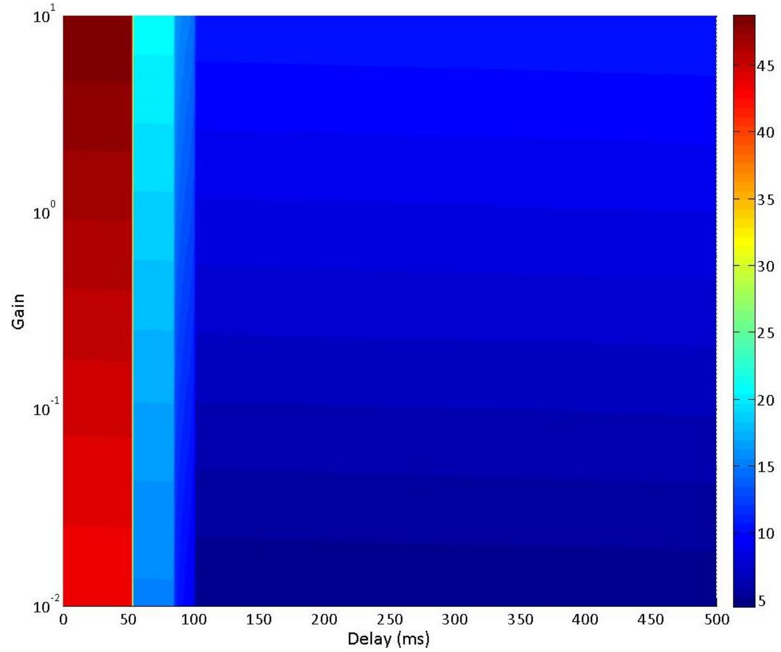


	CP	MC/CATD
Normalized Mismatch	0.6216	1.0396
Linear Gain	1.9658	0.7165
Delay (ms)	177.0833	277.0833

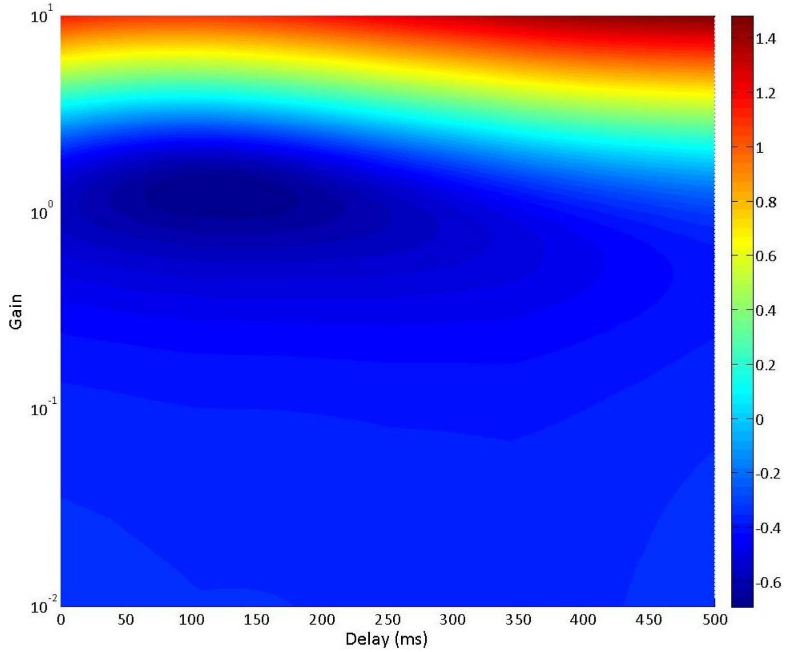
	CP	MC/CATD
Max. Correlation	0.6159	0.5307
Linear Gain	1.9792	0.7156
Delay (ms)	166.6667	272.9167

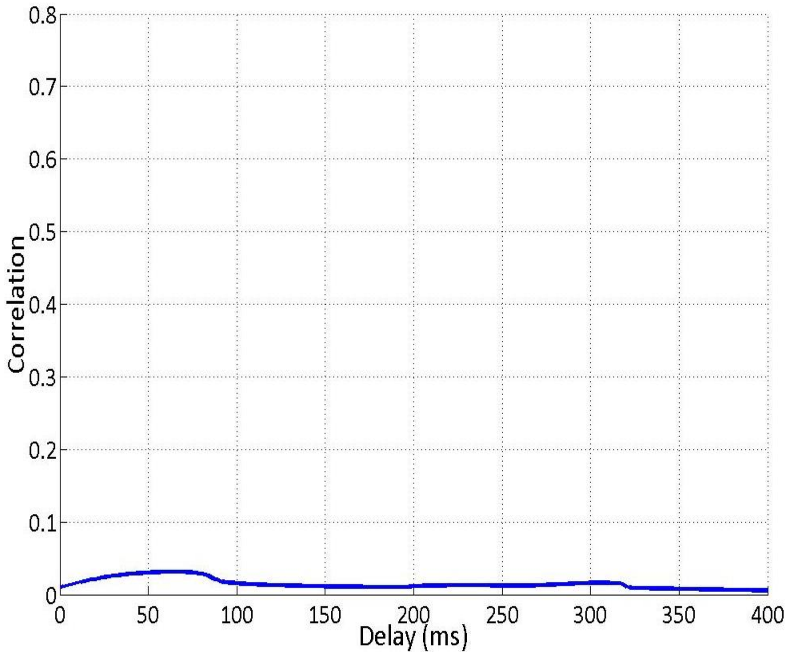
Bat Flight – Analysis of Steering Control – Bat-Insect Interaction

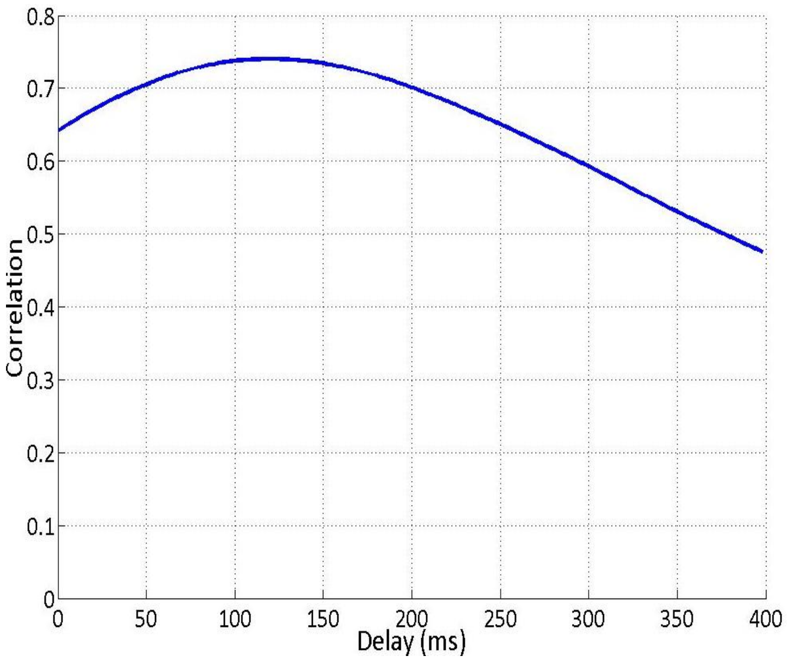
CP



MC/CATD

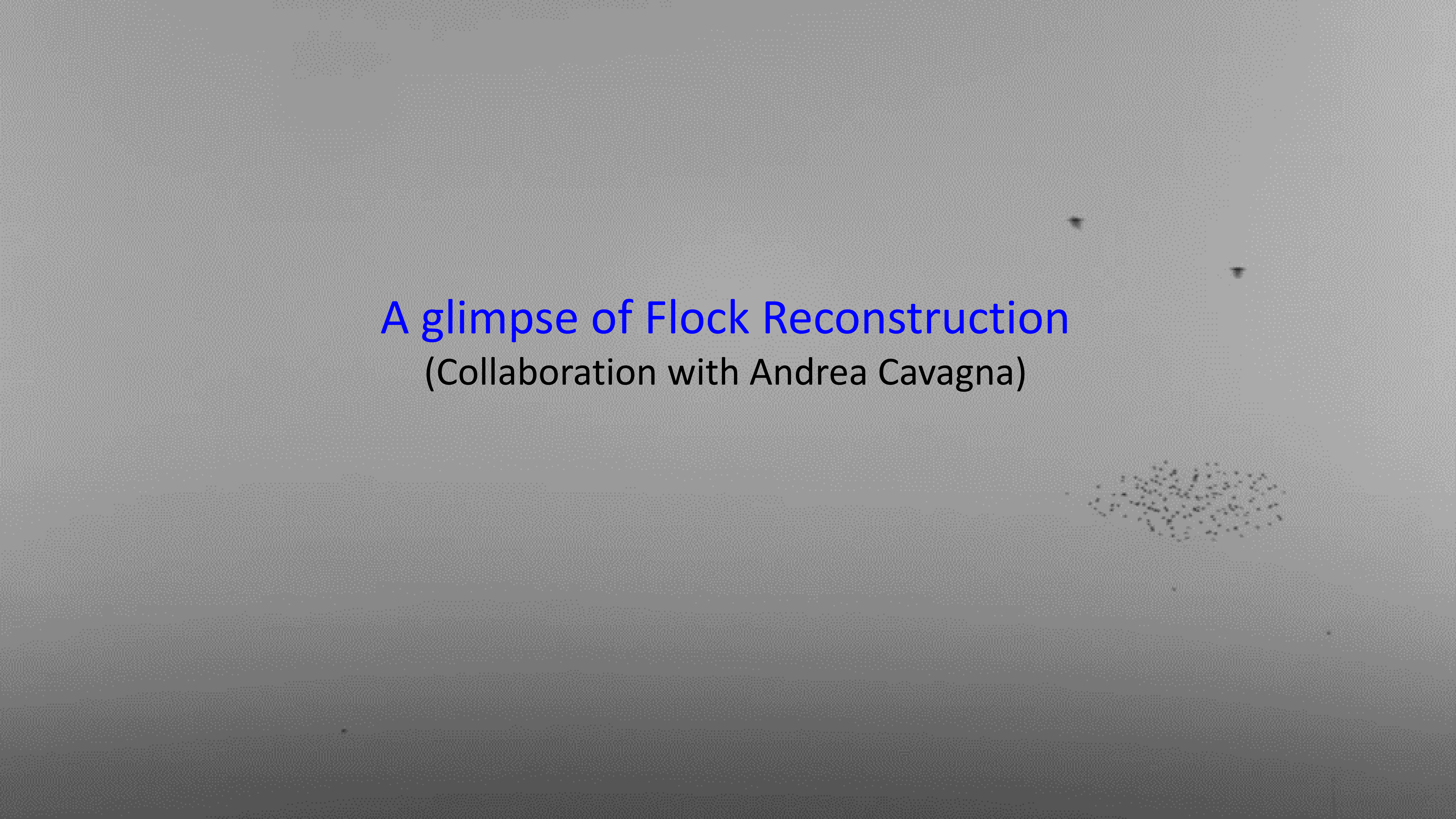






	CP	MC/CATD
Normalized Mismatch	7.6092×10^3	0.0511
Linear Gain	0.0100	1.2457
Delay (ms)	102	118

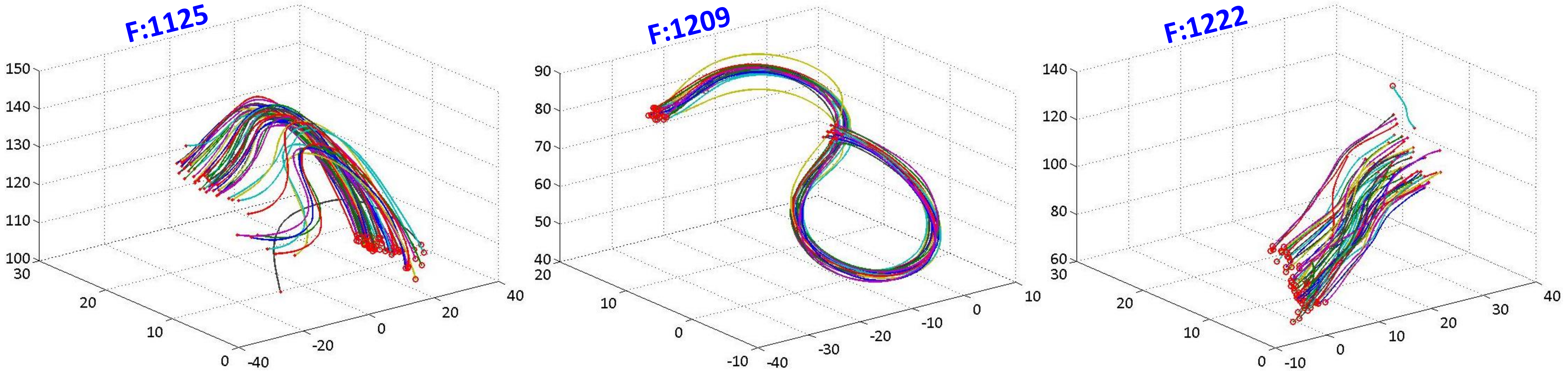
	CP	MC/CATD
Max. Correlation	0.0314	0.7403
Linear Gain	8.2354×10^{-7}	1.2457
Delay (ms)	64	120

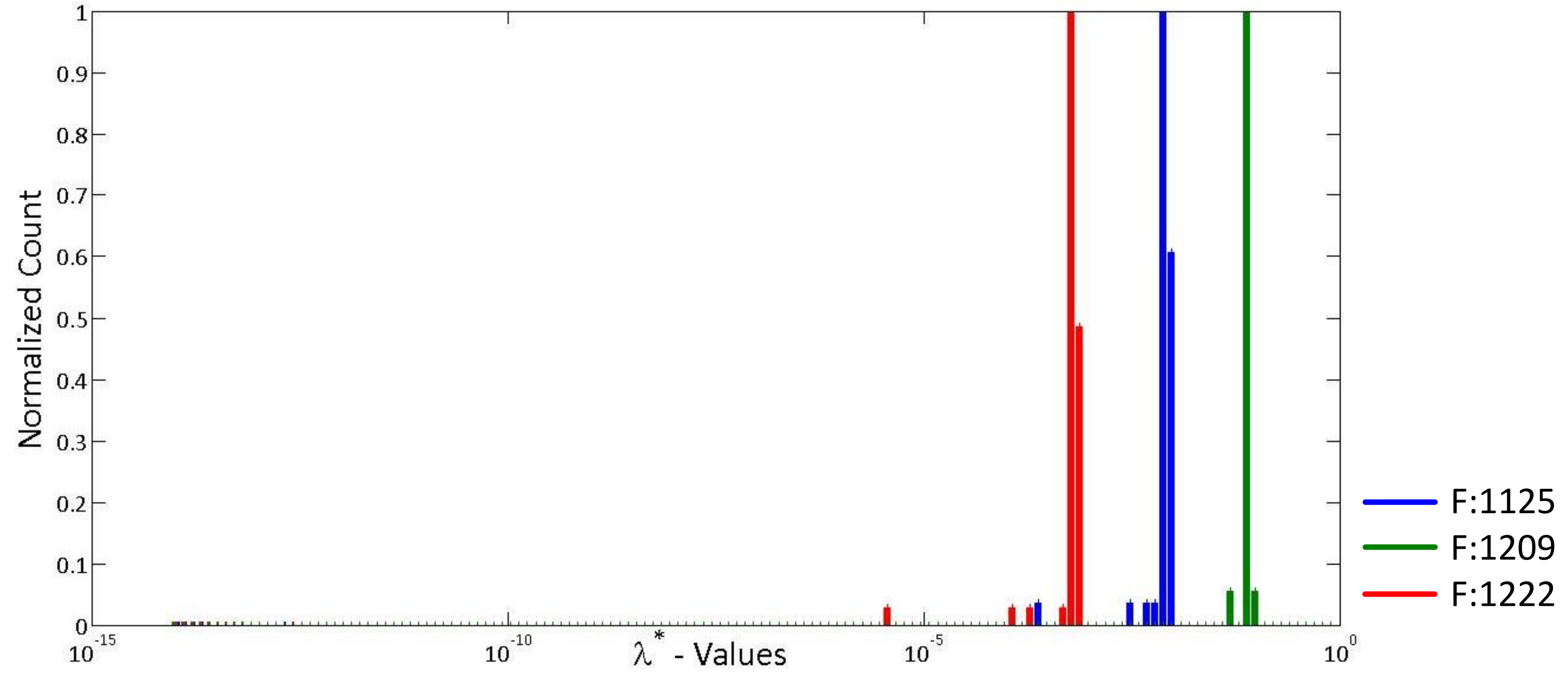


A glimpse of Flock Reconstruction

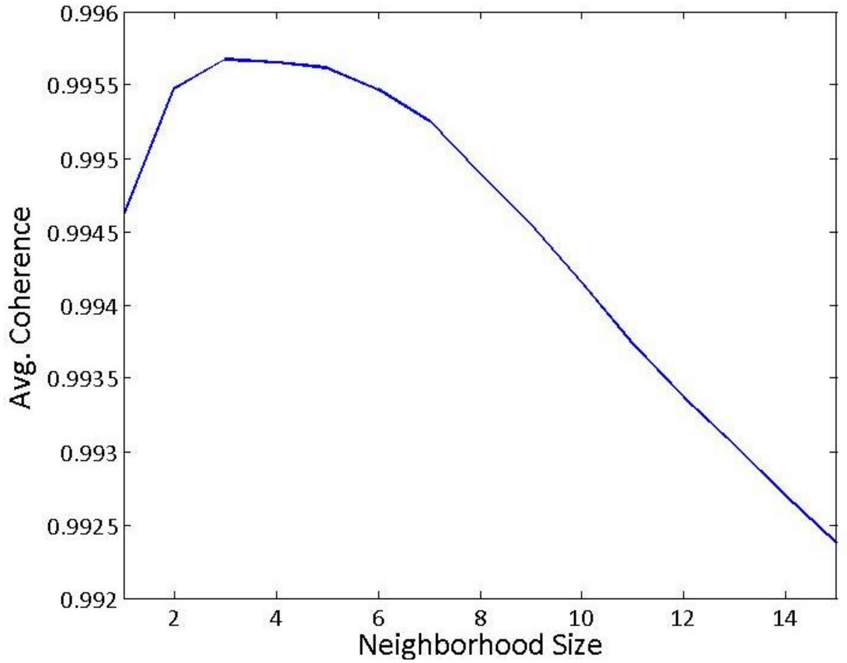
(Collaboration with Andrea Cavagna)

Analysis of Flight Strategy for a Starling Flock – Trajectory Reconstruction





Starling Flock – Flight Strategy



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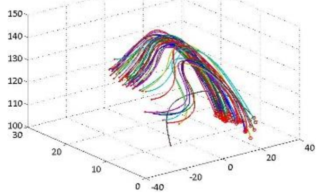

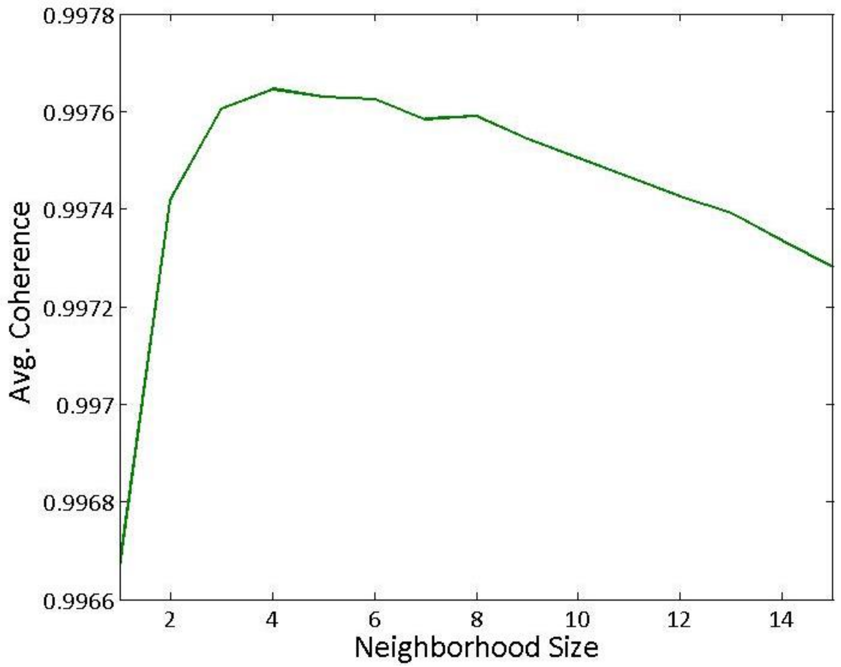

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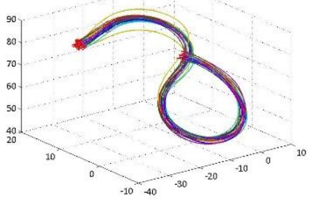

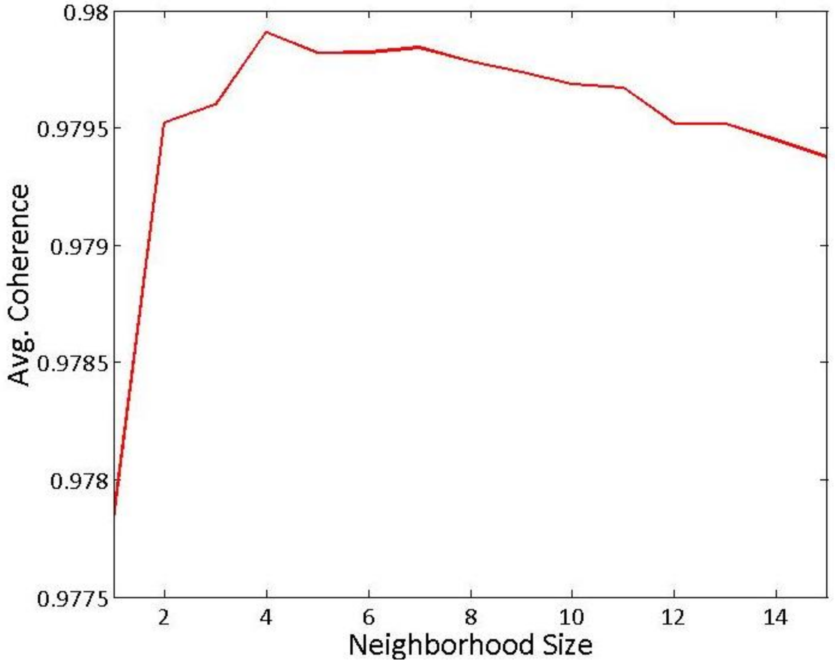

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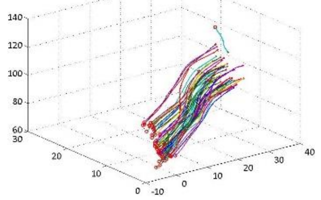
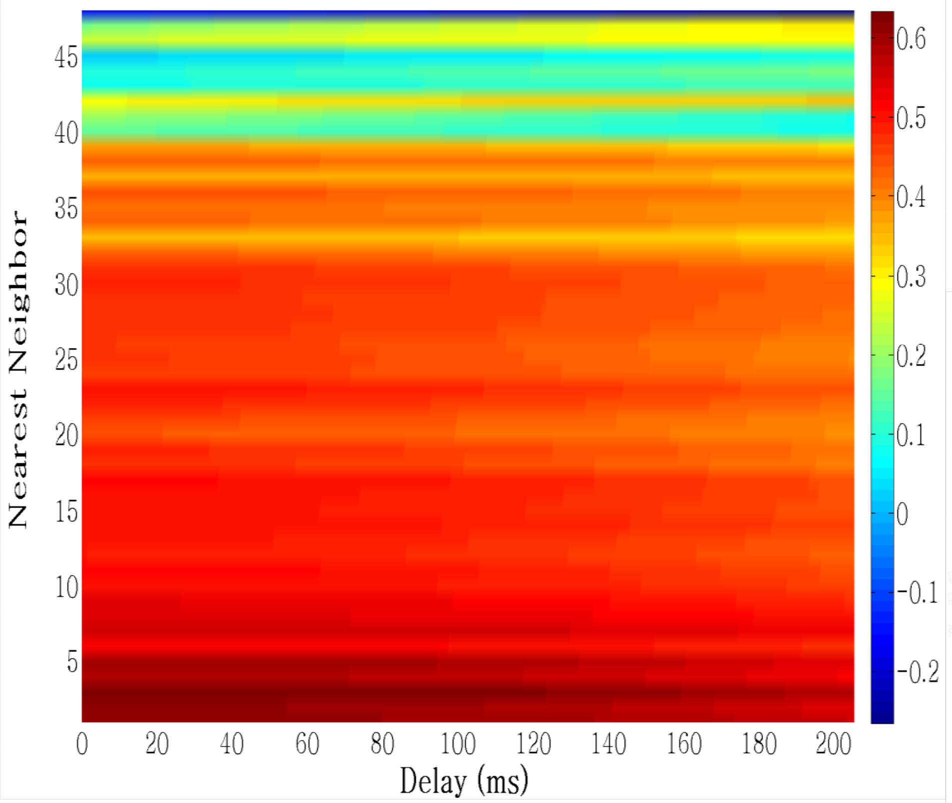


Starling Flock – Importance of Neighborhood and Delay



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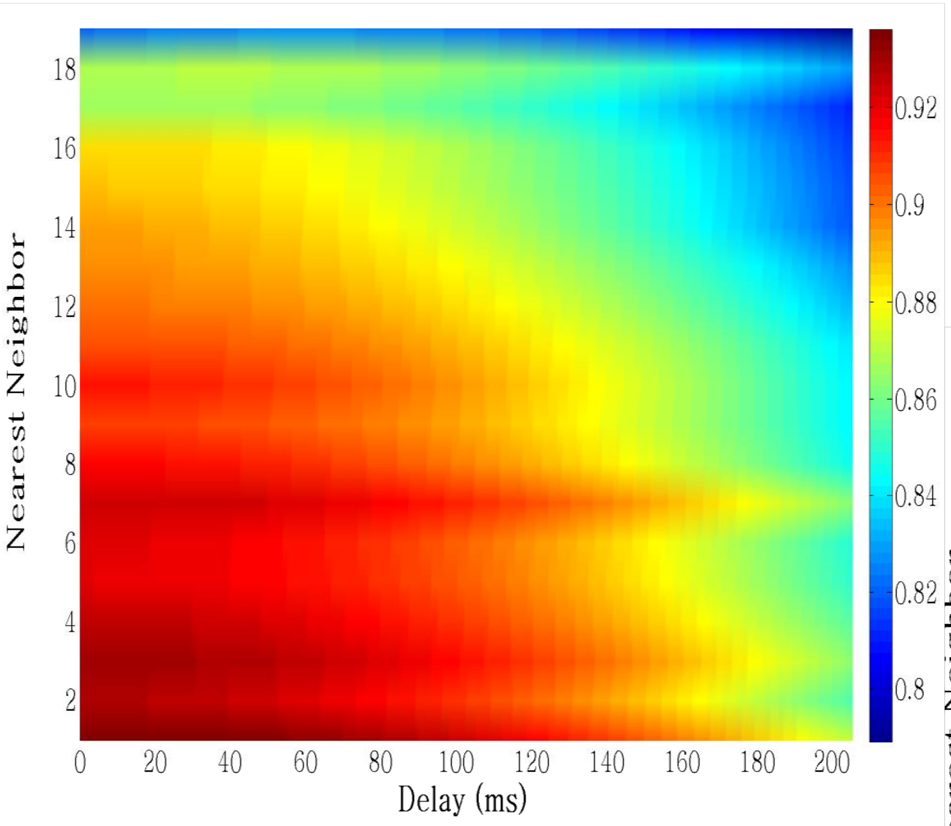

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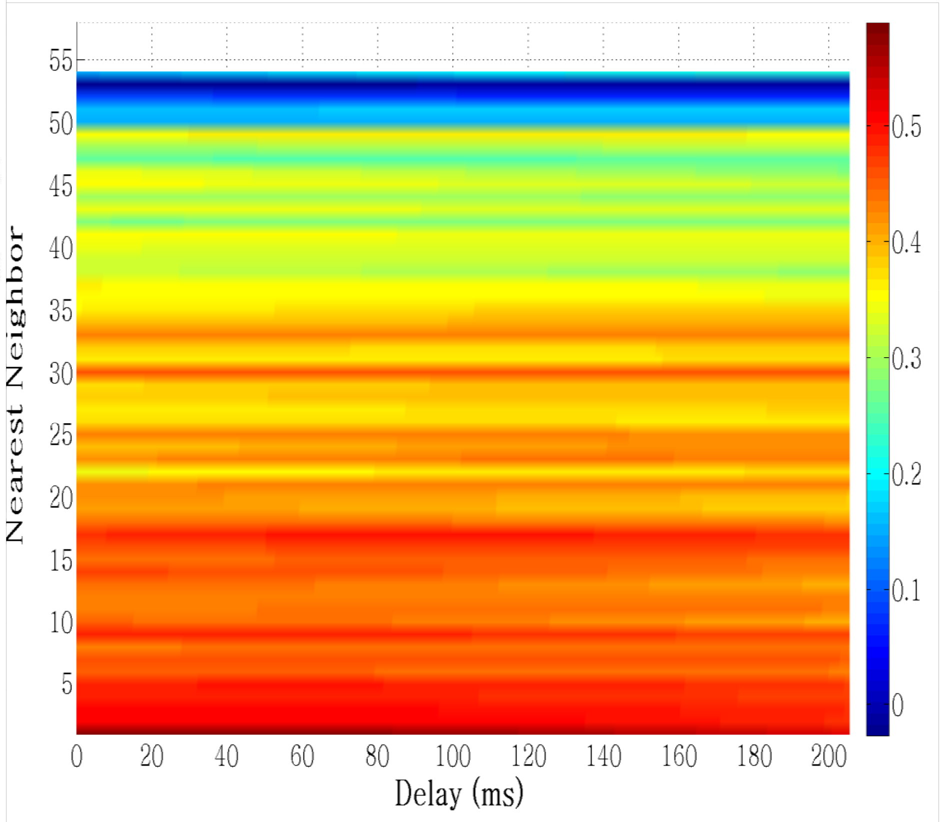

F:1222

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Thanks